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ABSTRACT

This book-length manuscript discusses some of the developments in mathematics, psychology and educational theory which have influenced new mathematics curricula. The projects are discussed in detail: (1) the OPI Mathematics Project begun in Budapest, Hungary; (2) the Sherbrooke Project in Quebec, Canada; and (3) the New York Project in New York City. Discussions of each project include brief historical comments, descriptions of the learning environments and vignettes of classroom discourse. Classroom objectives and procedures are related to the theories of Dienes, Piaget, and others. A chapter on evaluation discusses the need to evaluate curriculum projects and the difficulties of making certain types of assessments. The evaluation of the three projects is discussed. Future trends in mathematics education are predicted. Appendices provide a listing of some current projects on the teaching of modern mathematics and descriptions of materials used in the three projects. (SD)

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THREE MATHEMATICS PROJECTS OF THE DIENES TYPE

by

Zoltan P. Dienes

Director of the Centre for Research in Psycho-Mathematics
University of Sherbrooke, Quebec, Canada

Sandor Klein

Assistant Professor of Psychology
Teacher Training and Pedagogical Institute
Technical University of Budapest, Hungary,

and

Rainer Lant

Assistant Professor of Mathematics
Memorial University
St. John's, Newfoundland, Canada.

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Preface

The content of this book ranges over experiences drawn from many countries, in particular from Hungary, Canada and the USA. It summarizes the progress which teaching method in mathematics has been able to make as the result of Dr. Dienes' development of a psychological theory for the effective communication of this subject to children.

Clearly the preparation of a work of this kind would not have been possible without the generous co-operation of the authorities who opened up their schools for innovation and experiment, or without the psychologists and teachers who worked with Dr. Dienes in person. The reports and studies which he and they prepared in close co-operation over many years have been synthesised and condensed into what we hope will prove an interesting account for the non-specialist reader. Should the latter wish to pursue the subject in greater depth they he will be able to obtain considerable assistance from the bibliography. While being far from comprehensive this is intended as a springboard for further study. Moreover, the authors would always be pleased to entertain the discussions of troublesome questions by correspondence.

Our deepest gratitude extends to all who directly or indirectly furthered the growth of a project, contributed information for our chapters or helped these chapters to get into print, In particular we must acknowledge our debt to Dr. Varga as initiator of the Hungarian project, to the principals of the Ecole Eymard in Sherbrooke and of the Fleming School in New York, to Dr. Klein's assistants at the Pedagogical Institute in Budapest, to the psychologists Ingrid Krause and John Williams in Sherbrooke, to Graham Cantieni for his photographs, and to Marie-France Kyle and her secretaries for putting the manuscript into its final form.

Zoltan Dienes
Sandor Klein
Rainer Lant

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INTRODUCTION

Everybody is talking about the "new mathematics". It is not generally known that what is described as "new" is some 150 or 200 years old, having taken a Russian sputnik to get mathematical ideas past the obdurate conservatism of the curriculum kings. A few years ago it was the fashion to start mathematics projects; new catchwords and clichés were rapidly being formulated in order to regiment the unsuspecting educational public into just another straight jacket, the difference being that teachers and children now monotonously began repeating polysyllabic words such as"associative principle"instead of chanting tables in unison. Behind all the "modern" drum-beating, and the shameless commercial scramble to get in first with the shoddiest but most saleable mathematics curriculum full of the new clichés, there has here and there some serious work been done. (See Appendix 1) How can the uninformed and unsuspecting public possibly tell the difference between a carefully researched, mathematically and psychologically sound curriculum and accompanying methodology, and the mathematically and psychologically half-baked commercial packages peddled round the world by profit-hungry publishers ?

It is intended to give the public just a few guidelines, not of course foolproof ones, whereby a discriminating searcher amongst the morass of new curricula and methods can be helped to make a somewhat more informed choice than he might have done without these guidelines.

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* Based largely on Dienes (1973).

Before we can suggest what these should be, some excursions into mathematical and psychological territories will be necessary. Since there are as yet no satisfactorily validated educational theories, there will be no need for any excursions into educational territories. These are largely unexplored, mainly because governments have been unwilling to finance any expeditions of any scope whatsoever to explore them.

Much research into the foundations of mathematics has been done in the past few decades, and the architecture of the foundations is becoming increasingly clear to those mathematicians who take the pains to read up the evidence. The knowledge of how fundamental mathematical concepts hang together are now beginning to have an effect on how mathematics educators think it should be taught to children. The connexion is still extremely tenuous, since mathematics continues to be regarded as a very exacting discipline, and on the whole those who concern themselves with educational problems find it difficult if not quite impossible to grapple with the simple task of apprehending mathematical truths expressed in mathematical literature. It is really the demands of industry and of the armed forces that have forced educators to rethink their educational ideas on the teaching of mathematics, and in this way the beginnings of an alliance between the two camps are being formed. Since education is not a scientific discipline, in the sense that physics, chemistry, and mathematics are scientific disciplines, the alliance is a somewhat curious one. Educationists on the whole rely on books, with a certain modicum of visual and other aids, and on a certain amount of discussion in grappling with the practical problems of introducing the new mathematical architecture into the classroom. The old, ossified centuries-old mathematics curricula which the "new mathematics" is supposed to replace are being taught according to very well defined and well established practices, known as "traditional teaching methods". These must have been developed over the decades and centuries through trial and error and thus reached some kind of practical equilibrium, producing a certain proportion of mathematicians, scientists, technicians and the like. But today these proportions are no longer adequate for manning a modern technological society. Hence all the hue and cry.

Intuitively the reformers have realized that with new curricula must come new methods. And this is where the educationalists could not help. The obvious next question is, whether the psychologists could help ? Upon looking at the abundance of "learning theories" in psychology, it soon becomes clear that very little of any of these theories is of any use to the teacher on the classroom floor. Learning theorists have been so far concerned with the kinds of learning that are extremely elementary, and so the psychological problems about how such complex behavior patterns as those that abound in the learning of mathematics can be explained became the urgent but wide open problems of a section of psychology ill equipped to meet the challenge. The net result is that now more and more psychologists are becoming interested in what they refer to as the higher cognitive processes (as opposed to "lower" ones such as the learning of strings of nonsense syllables). Contacts have been established between mathematicians, educators and psychologists (see e.g. Dienes-ISGML, 1966), but there are still too few persons available who are equally well conversant with the disciplines of mathematics and of psychology, and at the same time familiar enough with the realities of the classroom to be able to contribute realistically to the eventual solution of the curriculum and method problem.

To make matters even more difficult for the layman, mathematicians are not agreed amongst themselves as to the desirability of teaching "structure" to young children. The educators and psychologists might argue with some force that the mathematicians are not entitled to any but a lay opinion on this subject. Worse still, psychologists are not agreed about the best methods of teaching children mathematics, and as most of them do not understand very much about the overall structure of mathematics, this is not surprising. The result is the free-for-all engaged in by everyone who can profitably fish in these troubled waters. Teachers as well as parents throw up their hands and the children get more and more confused.

In the three school projects around which the present text centres an attempt has been made to bring psychology, education and mathematics together in a new setting. Judged against the world's school population this is a small start, but nevertheless one which has given the authors the conviction that it was worth making and could lead to a satisfactory solution of the problem of innumeracy on a much larger scale in the future.

CHAPTER 1

NEW MATHEMATICS PROGRAMMES*

1.1 The Mathematical Picture.

Since the late fifties, a certain number of meetings of mathematicians, educators and psychologists have taken place on the international scene as well as within the framework of national cultures. During these discussions, the argument about teaching children structure and not rote-learned responses to certain sets of stimuli, has been practically universal and unanimous (Cambridge Conference on school mathematics, 1963). It may be assumed then that the general trend of educational thinking is in this direction. Whether this will eventually prove to be a success or not, it is not possible to say. Now what are these structures? In a short note it is impossible to give an account of a subject so vast, but the following flow chart (Figure 1-1) suggests the first fundamental structures with which a child might be greeted during his first few years at school.

Insert Figure 1-1 about here

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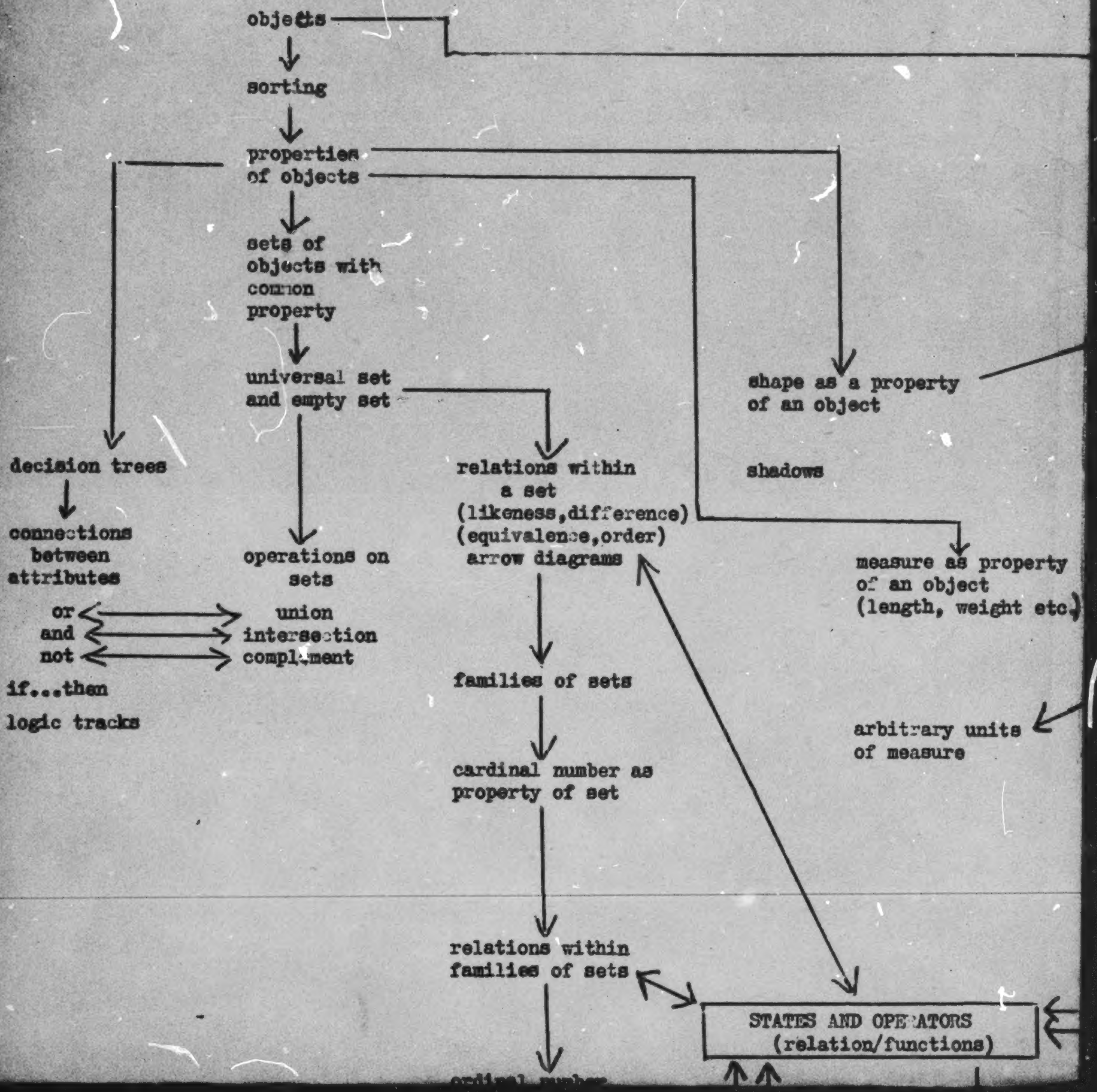
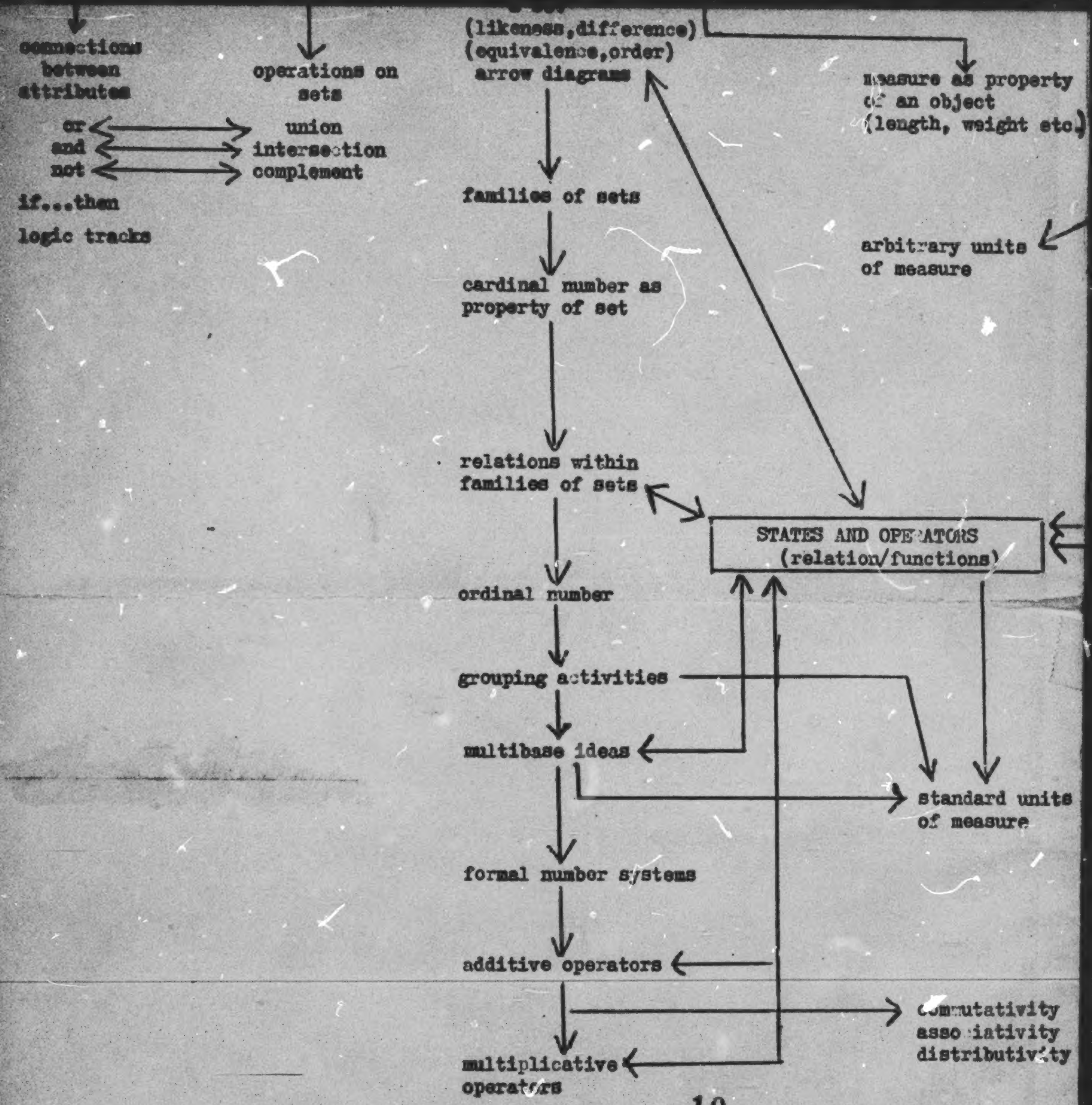
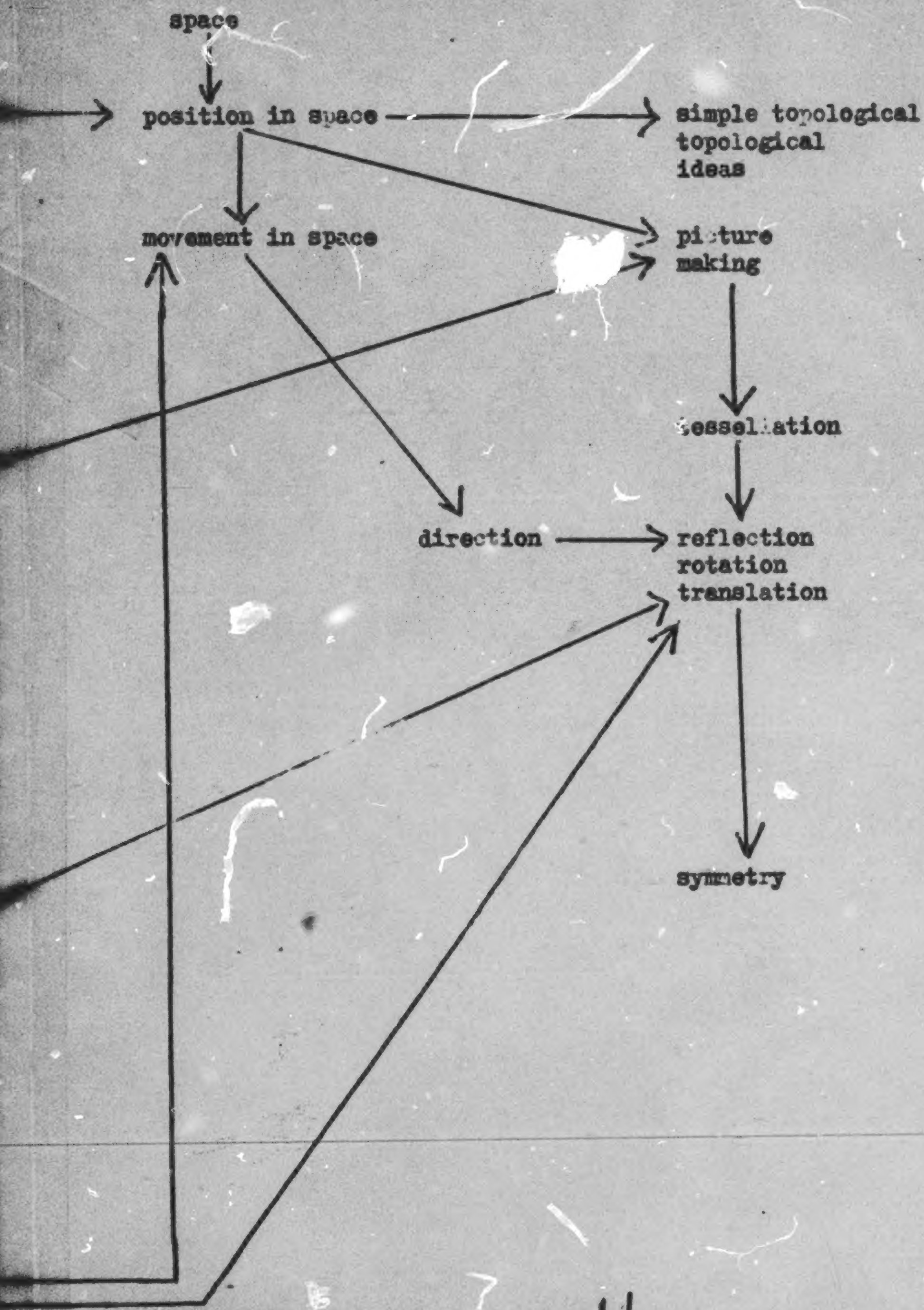
* Based largely on Dienes (1973b).

A FLOW CHART


objects
sorting
properties of objects
sets of objects with common property
universal set and empty set
decision trees
connections between attributes
or
and
not
if...then
logic tracks
operations on sets
union
intersection
complement
relations within a set
(likeness,difference)
(equivalence,order)
arrow diagrams
families of sets
cardinal number as property of set
relations within families of sets
ordinal number
shape as a property of an object
shadows
measure as property of an object
(length, weight etc.)
arbitrary units of measure
STATES AND OPERATORS
(relation/functions)


connections between attributes
operations on sets
(likeness,difference)
(equivalence,order)
arrow diagrams
measure as property of an object (length, weight etc)
or
and
not
union
intersection
complement
if...then
logic tracks
families of sets
arbitrary units of measure
cardinal number as property of set
relations within families of sets
STATES AND OPERATORS (relation/functions)
ordinal number
grouping activities
multibase ideas
standard units of measure
formal number systems
additive operators
commutativity
associativity
distributivity
multiplicative operators


space
position in space
simple topological
topological
ideas
movement in space
picture
making
tessellation
direction
reflection
rotation
translation
symmetry

The main path in the structural map above takes care of the journey as far as the natural numbers are concerned. The operators of adding and subtracting on the one hand, and those of multiplying and dividing on the other, offer suitable growth-points for extending the number system to integers and fractions respectively.

Considering successions of adding and subtracting operators, it is always possible to reduce such successions to either one addition and no subtractions, or to one subtraction and no additions. Such are the considerations that lead to the extension of natural numbers to positive and negative integers. Again, we can consider successions of multiplications and divisions. Any such succession can always be reduced to a pair of such operations, one multiplication and one division. This then leads to the introduction of rational numbers or fractions. The neutral operators in these two cases are "add or subtract nothing", known as zero, and "multiply and/or divide by one" known as one. The necessity to have states on which any of these operators can operate leads to the introduction of integer states, i.e. states of "minus one", "plus three" and so on, as well as to the introduction of fractional states such as "two thirds", which for example is the name given to the state arising out of operating on the state one by a multiplier 2, followed by a divider 3 (or in the reverse order). "Plus three" is the name of the state obtained by operating on the zero state by the "add three" operator. "Minus one" is the name of the state obtained by operating on the zero state by the "subtract one" operator and so on. The combination of the two extensions give us all the positive and negative rational numbers.

In the study of movements in space there are corresponding operations, with almost identical structures. If we move ourselves in a movement which takes every point of our body along in a straight line, i.e. if we never turn, we perform a translation. Translations can be "added", since the effect of any two successive translations can be brought about by one single translation. "Subtracting" a translation could be the adding of that translation but in the opposite direction. It will be verified that all the rules of adding and subtracting positive and negative numbers are obeyed by such translations.

There are also spatial equivalents to multiplication and division. These are enlargements and shrinkings. Shine a flashlight on to a plane figure, and then let the shadow fall on to another plane which you hold parallel to the plane of our figure: An enlarged version of the figure will be seen. If the distance from the flashlight of the plane of the shadow is to the distance from the flashlight of the plane of the figure as q is to p, then for every distance of p units in the figure there will be a distance of q units in the corresponding part of the shadow. This means an operator $\times \frac{q}{p}$ applied to the state of the figure will give another figure. It is possible to reconstruct the shadow figure out of some material such as cardboard, and apply to it a further transformation, say $\times \frac{s}{r}$, so that for every distance r in the second figure there should be generated a corresponding distance s in the third figure. The problem of generating the third figure from the first one will yield as a result the multiplication rule for fractions.

The above are merely some examples of how the arrows on the structure-diagram can be turned into realities. Many mathematics programs, particularly as they are put into texts, seem to degenerate into a series of topics, without much attempt being made to connect the topics in any natural way.

Examples of structuring the mathematics programme with a view to emphasizing connexions could be multiplied indefinitely. It is a pity that more designers of mathematics curricula do not make such structuring one of the aims of their programme.

One other important feature of any mathematical structure might be pointed out. Apart from the connexions of a structure with other structures, discussed above, we might be interested in the generalizability of the structure, or even in ways in which a structure can be presented in a way which is as general as possible and yet within easy grasp of a child's comprehension. Much of the fertility of mathematical thought comes from the mathematicians inveterate habit of trying to generalize, or to look at things more generally. One way to help the achievement of generality is to vary all possible variables inherent in a mathematical structure. For example, a space is not just one-dimensional for a mathematician; it can have any number of dimensions. Linear operations are extended to non-linear ones, rotations about one particular axis to rotations about any axis,
and symmetries about a line in a plane are extended to symmetries about a plane in space. Those responsible for either the construction or for the application of curricula could well bear such considerations in mind and see whether there are any encouragements for children to generalize as they go through the curriculum, or whether the "topics" are presented in the most general ways possible. For example, is arithmetic restricted to "tens and units", very slowly graduating to "hundreds" and so on ? How can a child generalize if the steps by which he has to make such a generalization are so few in number and so far separated in time ? And if he is given only the base of ten, how can he get the taste for generalizing ? If we close up artificially the territory he can move in, how do we eventually expect him even to want to

get out of it, let alone succeed in doing so ? If we give him one or possibly two dimensions of geometry to work in how will he ever think of an arbitrary number of dimensions (variables) ? Seeing that the generality of structures should be met with as early as possible, in order to prevent the child from making a premature closure around a particularized structure the opening of this closure, i.e. generalization, would be more difficult at a later stage. The study of number bases as a mathematical tit-bit at the age of eleven will not have the same effect as the study of the expression of number properties in any arbitrary base at the very beginning, i.e. round about the age of six.

1.2 The Psychological Picture.

Another consideration in assessing a curriculum is its treatment of the role of symbols in mathematics. For those people for whom mathematics is a meaningless empty shell of manipulative rules, mathematics can indeed be equated to the symbols by means of which it is expressed. But to those of us who regard mathematics as abstracted structure, the symbols are but the purveyors of such a system of structures. In this case we must surely be careful not to confuse the process of abstraction with the process of symbolization. If the symbol is to stand for an abstract structure, then this abstract structure must have been in some way formed or at least begun to be formed if it is to be possible for the symbol to stand for it. In assessing the way a programme of mathematics is presented it is as well to examine whether any attempt has been made to consider the processes of abstraction and of symbolization as separate though related. As far as abstraction is concerned the question whether a particular curriculum takes steps to ensure that abstraction does in fact take place is of crucial importance. Since an act of abstraction is an act of realizing the common feature of many different situations, one would expect those who construct curricula and their accompanying methodology to have at least attempted to present mathematical structures in a number of different ways.

To attempt to apply an analytical type of technique to the learning of structures that have not been properly constructed is likely to be ineffective from the point of view of both comprehension and efficiency in handling. Construction (mental as well as physical) takes place out of the component pieces, with a view to putting them together to meet with certain kinds of requirements. This is always the preparatory stage to the establishment of the insight about the internal relationships of the pieces, with possibly a few exceptions. Care should be taken that the organic nature of the concept formulation cycles is not unduly interfered with. Since different children will have different dynamics of constructing their concepts, provision should be made for individual differences. This can be done in various ways. The children may be grouped and encouraged to work from instruction cards. Alternatively, they may be put to work with a more severely structured type of programmed text. The latter, however, seems to take the life out of mathematics learning as a dynamic and exciting adventure (see e.g. Dienes 1964, Dienes-Golding 1967).

Where possible counterexamples should always be given to illustrate cases in which conditions in a theorem are not met and its conclusions can not apply. In the beginning when notions such as commutativity and associativity are being learned, it would seem sufficient to present children with some non-commutative and some non-associative situations, making these as natural as possible; this introduces them to contrast. Certain things happen sometimes and the opposite kinds of things happen at other times.

1.3 Points to look for in Mathematics Curricula.

It is now possible to give some pointers, possibly helpful to the users of the many variegated mathematics programmes and curricula, to enable them to make a less random judgement than they would otherwise have done. Our judgement, based on the considerations of the last few paragraphs, will naturally be biased. It will be biased heavily in favor of structure and against associationism, in favour of children's liberty of thought and against authoritarianism, and in many other ways which will be quite evident to the reader. If the reader should find this bias unacceptable, he is invited to compile a more satisfactory set of principles according to which a different set of suggestions could be constructed.

Here is a questionnaire in which the underlined answers are biased towards the kind of dynamic and psychologically viable mathematics programme as conceived by us. A score can be obtained for any programme by simply counting the number of answers which are the underlined ones when applied to the programme in question. The questionnaire includes questions of a general, mathematical and psychological type.

1. Does the programme mainly serve future industrial needs ?

Yes or No

2. Do "modern words" such as set, commutativity, associativity or group mostly occur in just the one part of the programme in which they are introduced ?

Yes or No

3. Are the instructions to the teachers so explicit as to give her no choice of action ?

Yes or No

4. Are there many references to other relevant mathematics programmes?

Yes or No

5. Are the connexions between parts of the programme stressed throughout ?

Yes or No

6. Does the programme proceed slowly through a very large number of very small steps ?

Yes or No

7. Are consistent attempts made to present structures in ways as general as possible ?

Yes or No

8. Does the learning of each structure begin with an attempt to construct, from concrete experiences at a young age and from mental ones later on ?

Yes or No

9. Are all children expected to cover each part of the programme at the same time ?

Yes or No

10. Are individual differences amongst children adequately catered for in the way the programme is organized ?

Yes or No

11. Are children presented with open ended situations as a regular feature of the programme ?

Yes or No

12. Is the process of abstraction encouraged by the presentation of the same structure in a number of different guises ?

Yes or No

13. Is there frequent use made of contrast in making clear the meanings of mathematical properties or of conditions of theorems ?

Yes or No

14. Are the ideas (i) symbolic, (ii) abstract, confused with each other in the development of the programme ?

Yes or No

15. Is provision made throughout for children to check the validity of their thinking ?

Yes or No

16. Is there an attempt to relate mathematical ideas to those in other fields such as language or music ?

Yes or No

It is suggested that a score of 10 or over is a good score, a score of 5 or under is a bad score. Again, the scoring can be altered according to the particular bias of the reader. For example a "correct" answer to question 5 might be given more than one point.

Projects also differ significantly from one another in other important ways, the rightness or wrongness of which is a far less clear cut matter. Some of these differences may be pinpointed by attempting to answer questions such as the following:

1. Which of the following does the project regard as important sources of involvement for the student ?
   - a) The teacher's personality
   - b) The environment
   - c) Structured materials
   - d) Texts and worksheets
   - e) Films
   - f) Sound and video tapes
   - g) Group work and co-operation
   - h) Competition

2. The nature of mathematics calls for some hierarchical ordering of its presentation. Does the project treat this ordering as;
   - a) A closed sequence of stepwise exposition, as in programmed learning ?
   - b) A sequence of prompts for investigations which lead the student to discover principles for himself and gradually put them into a logical order ?

3. Any group of mathematics students, however homogeneously composed, will sooner or later reveal a distribution of ability.
   Does the project
   - a) Adjust its approaches to meet this distribution ?
   - b) Provide a minimal sequence, closed or otherwise, which the weakest students are able to complete ?

4. Does the project
   - a) Believe in keeping the students of a class together by periodic evaluations ?
   - b) Allow the individual student to assimilate sequences at his own rate and take an evaluation when he is ready ?

5. Does the project
   - a) Use the environment to promote the students' general appreciation of the value of mathematics ?
   - b) Use the environment to further the abstraction of mathematical structures ?
   - c) Use the environment for the application of mathematical structures which are already known ?

6. Does the project use modern topics
   - a) In isolation from traditional topics ?
   - b) In conjunction with traditional topics wherever this is appropriate ?
   - c) For their psychological value in helping to develop powers of abstraction, of generalization and of their mathematical mental process ?

7. Does the project in the teaching of a given topic
   - a) Set a deliberate limit to the choice of possible approaches ?
   - b) Liberally borrow from what other projects have to offer ?
   - c) Try to select approaches to suit the individual student ?

Our two questionnaires are designed to provide the reader with a set of ready made criteria for assessing the three projects which are about to be described and for comparing them with one or more of the projects that have tried to solve or reduce the learning problems of children in the elementary school.

CHAPTER 2

The OPI Mathematics Project in Hungary

2.1 Origins and Evolution

After about two years of preparation (Varga 1964), in September 1963 a school mathematics project was started in two classes of a school in Budapest, Hungary. Initiated by Tamas Varga, the project has since been developed into a nationwide scheme, the OPI Mathematics Project, spreading at present over one hundred classes in a dozen Hungarian cities. The OPI Project basically set itself the same task - but with much more limited opportunity - as the Madison Project, that of "Seeking the best experience with mathematics which can be provided for children at the pre-college level" (Davis 1964).* Its start is well described by Varga himself:

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* Though the OPI Project developed mainly from the concepts of Dienes, it has also assimilated certain ideas from educationists working in the same direction all over the world: Sawyer in the United States, Davidov and Elkonyin in the Soviet Union, Krygowska in Poland, and others.

2.2 First beginnings

"Pilot work in the teaching of mathematics first started in Budapest in the autumn of 1961. After a two weeks' summer course, many teachers expressed their agreement and sympathy to the new approach. But when it came to the question of doing some teaching based on the same principles, each of them had some excuse. They clearly found the new ideas dissimilar to anything they had experienced before. Thus, we had no teacher at our disposal. We did have two sets of Multi-base Arithmetic Blocks, a fair amount of ideas about what to do, and the eye of many people on us (not all of them kindly disposed to the announced experimental teaching). Should we retire? I decided to begin myself with a class of 36 seven-year-old girls -- an age group totally outside my earlier teaching experiences. The authorities were far from being enthusiastic about the project. Eventually I was permitted to teach extracurricular material (especially non-decimal arithmetic based on the M.A.B.), but it was made clear that the official syllabus had to be covered as well. The class teacher, a self-conscious lady particularly proud of her methods of teaching arithmetic, was ordered to be present at my lessons without ever interfering. There she sat the entire year, and tried not to smile at my blunderings with children who were more favorably disposed than she. They liked playing games instead of waiting their turn to answer a question posed by the teacher. They liked the freedom to leave their places (in order to change material or worksheets). They definitely preferred this to sitting immovably on their seats. They also liked changing their ideas during the games without being punished for breach of discipline. This freedom was permitted them only three times a week, each time for a period of about eighty minutes. They availed themselves of the opportunity eagerly. The class teacher deemed that the freedom was excessive and that this jeopardized her educational efforts during the rest of the lessons. I could not doubt the veracity of her statement; very different educational practices being used in close juxtaposition are always likely to lead to conflicts.

The result of this first trial, after the first year of teaching, was the refusal of the authorities to give permission for another. The children, and a number of parents, were sorry. The class teacher took notice of the decision with a certain relief. To be frank, so did I."

A New Start

"In the 1962-63 school year, the M.A.B. material served mainly as a plaything for adults. During this time, however, the climate regarding non-standard teaching experiments in mathematics underwent a considerable change. This was mainly due to the International Symposium on Mathematics Teaching held in Budapest (27 August - 8 September, 1962). It had to be realized that the usual slogan "The teaching experiences of our best teachers should be diffused and generalized" was no longer sufficient in our age of rapid changes and that radically new departures had to be made. I was invited to start similar work again, this time in two first grades (6 plus year olds) of another school. My duty was to design the syllabus and the teaching materials, also to instruct and supervise the teachers.

This was a great opportunity and I felt I had to avoid taking too much risk. I decided therefore not to impose anything upon the teachers. They had to be convinced of the potential usefulness of what they were going to do and never to jump into darkness. Both teachers had much good will and they were well qualified. Neither of them clung rigidly to her views concerning either the syllabus or the teaching methods but they would not have given up willingly their feeling of security developed during their years of school teaching. They were instructed by word of mouth and by various written materials, including relevant chapters from Z.P. Dienes' Building Up Mathematics and L.G.W. Sealey's Creative Use of Mathematics in Junior Schools. Various efforts were directed towards enabling them to visit Leicestershire -- where much the same sort of learning as we were trying to establish had been initiated in many schools. They have not been successful as yet, though it is likely to be realized next year. The opportunity to visit these schools was, however, granted to me. I spent one month studying the work of the Leicestershire Mathematics Project by the courtesy of the local educational authorities."

The Syllabus

The traditional first grade syllabus in Hungarian schools covers numeration (only decimal, of course) up to one hundred, addition and subtraction up to twenty, with simple problems on the different aspects of these operations, the use of the " = " symbol in the sense "gives as result" and some weights and measures. As explained earlier, I had to be cautious in not departing from the usual syllabus too much; this is especially important for the first year. Striving for greater generality was the main aspect of the alterations:

1. Children were led through appropriate exercises to the grasp of " = " as a symbol meaning "the same number as".

2. "Equal" was a border case for them between "greater" and "less". They used the symbols " > " and " < " before they got acquainted with the symbol " = ".

3. By the use of Multibase Arithmetic Blocks they were led towards the general idea of place value notation rather than just to the decimal notation as a special case.

4. Through the use of Cuisenaire rods, their attention was directed toward some aspects of addition and subtraction independent of symbolism and of the particular numbers involved.

The work with these two classes was to be continued for at least three more years, and more venturesome digressions from the official syllabus were planned for these years. On the whole, without having broken away from the teaching tradition of their own and of their fellows, our two teachers gradually became aware that methods quite different from the accustomed ones are conceivable and, perhaps, even preferable. Consciously or not, they also began to appreciate some similarity between their relation to the children and mine to them, both of which can be characterized by Polya's words: "Suggest it, do not force it down their throats."

2.3 Further development

The organisers of the OPI Mathematics Project emphasized their opposition to a rapid, unfounded development. Four years later, Varga (1968) continued to write:

> "Gradual extension is a key word for us. We are against conclusions of the kind that " The results of our experiments show that our way of teaching mathematics is better than the usual, therefore it should be introduced into mass education by the school-year n/(n+1)!" By what reason and right would we impose the reform on teachers if we are against imposing mathematics on children? What we would like is:
> 1. to acquaint as many teachers as possible, as deeply as possible, with our way of teaching mathematics, or rather with some version of it, and
> 2. to attain that every teacher be given the opportunity (including the right and material aid) to practice an up-to-date teaching of up-to-date mathematics."

In spite of this, the quantitative development of the Project was impressive

The latest development reveals that the OPI Mathematics Project received permission from the Ministry of Education to introduce this method through to 1979 in every Hungarian school where the conditions are satisfactory.

2.4 Principles and objectives

The philosophy adopted by the OPI Mathematics Project may be summed up as "A break with the usual fragmented approach, and a view of mathematics as a whole."

Every item in the traditional curriculum is queried and a basically new set-up suggested. Ideas traditionally belonging to algebra and delayed until Grade 8 are suggested and largely adopted as early as in Grade 1, and developed according to children's abilities and needs. Many key ideas of mathematics, some of which are quite unusual in the primary (or even secondary) grades, appear in the OPI method in the first few years.

Algebra and Geometry are just as well represented in the curriculum as elementary ideas of topology, information theory, set theory, logic, probability and statistics, all at the level of children, six, seven, eight, or nine years of age.

The criteria on which the selection of the subject matter is based are the following:

1) The subject matter must be fundamentally important from the point of view of the whole of mathematics. For example: equations, inequalities, sets, functions with tables or functions from graphs, functions with formulae are all fundamentally important mathematical ideas.

2) The subject matter should be easily accessible to children through problems and through play. If an entry to the subject matter could be arranged through the use of the children's tactile sense, we have always considered this an advantage although not as an absolute condition.

3) The subject areas must be in intimate relation with each other and they must reinforce each other and thus help to construct an organically developing system.

4) The subject areas to be introduced are to prepare and to introduce the teaching of other important subject areas.

5) The subject areas must have use in modern applications of mathematics. For example, for the understanding of the functioning of electronic computers, it is absolutely necessary to be familiar with systems of numeration different from that based on 10.

Varga continues:

> "It goes without saying that we believe in the so-called spiral curriculum. Every subject matter, which might have been brought down from high school or from the university to our first and second grade children, is regarded as a first stage leading towards a more rigorous treatment later. It must also be realized that we include the ordinary mathematical activities such as the learning of multiplication tables. As a matter of fact, our children seem not so much to learn multiplication tables as to discover them. So they find the process more pleasant. We have not placed a great deal of emphasis on the improvement of fluency in giving answers to multiplication facts. We have placed a great emphasis on the extent of children's ability to work things out, to reason and to discover relationships. We feel that after such preparation, the further pieces of mathematics can be based on a much more solid foundation. In the traditional elementary school only the arithmetical operations together with

a few questions on mensuration and some verbal examples are required. This is a gross under-estimation of children's abilities at the elementary level. It is clear to all who have studied the problem that a great deal of change will have to be made in the elementary mathematics curriculum. It is becoming more and more certain that such changes, at any rate at the elementary level, will be impossible without considerable and fundamental changes in methodology. The main issue is to make mathematics sufficiently challenging for children, so that it is worthy of a thinking effort. Sonner or later this pays dividends. If there is no thinking effort, the memory is likely to be over-burdened. We might lighten the burden of the student by giving him more thinking to do, as it will save him from remembering too much. But to act in this way we must change both the curriculum and the method.

Nobody need fear for our experimental guinea pigs, nor worry lest we drain the last ounce of energy from them in order to squeeze out some experimental results. They get practically no homework. They feel that during the lesson they are merely playing. In a sense, of course, they are right, although if one looked at the curriculum,one might be tempted to feel that the children were terribly overburdened. The curriculum includes such topics as arithmetical operations in the binary, ternary, four base, five base, and six base systems which are performed with ease by our young guinea pigs. It is often considered that such topics are difficult even at the secondary stage. To make matters worse, they use very long numerals, consisting of five, six or even more digits. They simplify fractions, they add, subtract, multiply and divide them, they solve equations with one or two unknowns, they work with inequalities, they examine tables of values for functions, they complete such tables, they learn representations of points in a plane through rectangular Cartesian coordinate systems and, in addition to all this shocking amount of work, the dreaded topic of set theory is introduced. But anyone who looked inside the classroom would immediately realize that no child there was anywhere near a mental breakdown."

2.5 No pressure on teachers or children

Teachers joining the scheme are given material and suggestions that may and do enable some of them to break radically with the traditional curriculum and old fashioned approaches. However, those who feel uneasy with too sudden a change are not urged to switch over at once. The OPI Mathematics Project advocates what Robert B. Davis of the Madison Project has called "light touch policy" in dealing with children.

This is especially important at the start when the teachers are teaching a great deal of material with which they have had no contact whatsoever before. " For example, the teachers have had to learn how to handle arithmetical operations in different bases of numeration. They had no previous instruction to prepare them in the experiment. The preparation has taken place on the classroom floor and the problems have been faced as they have arisen. Both the curriculum and the methodology were new to them. We adopted, for the start, a permissive attitude towards the teachers. Any new activity about which they felt uncertain they were advised not to engage in until they were certain. This permissive attitude has resulted in eventual full cooperation from the teachers who have taken an active part not only in the execution but even in the planning of the program. The positive attitudes towards mathematics which children developed very quickly as a result of the new methods went a long way to persuade teachers of the efficiency of the new methods " (Varga, 1965).

2.6 Abstraction the interiorization of concrete experience

The implications of Jean Piaget's developmental psychology for mathematical education are taken into consideration essentially as interpreted by Z.P. Dienes (1959; 1961). Varga (1965) writes:

> " In our experiment perhaps the primary educational principle which we have been using is that the process of abstraction is an essential one for which considerable time must be allowed. Children do not abstract automatically.
>
> We believe that it is only possible to abstract from the concrete, and in order to abstract, the child will have to get familiar with a great number of situations. Mathematics is very abstract. This is in fact its greatest strength, since it means, in effect, that it condenses into itself the essence of a great number of concrete phenomena. In order to get at this very abstract structure, we have to start at a very concrete starting point and lead children towards abstraction by making sure that a sufficient number of varied concrete experiences are provided for them.
>
> At the initial stages very concrete experience means sensori-motor experience. Tactile experiences are most important. In this connection our principle of abstraction implies that operations with symbols must be preceded by operations with objects. But, of course, there must be a two-way traffic between objects and symbols. It is also necessary for children to know how to interpret symbolic work, applying it to the object world. Although initially the activity is from object to symbol, later the

activity from symbols to objects must also be encouraged.

This is, in a way, what Piaget means by interiorization. All this means, in practice, is that we are putting tools in the children's hands. A distinction must be made, however, between tools used by the children and tools used by the teachers for demonstration purposes. Demonstration tools are for showing children certain relationships. The use of a working tool by each child insures that each child, as far as possible, will discover these relationships for himself."

2.7 Independence training

It is suggested that children should be allowed to make their own choice of learning activity and depth of problem solving, that they should be encouraged to construct their own problems, and control their own interaction with other children. A variety of ways of organizing the class contributes to this end. Individual work, or work in small groups with concrete materials and/or worksheets leads them to independent discoveries; seminar-like discussion led by the teacher with the whole class or a part of it can either initiate new ideas or contribute to consolidating and systematizing what has been achieved so far.

Independence training is an important tool in the OPI Mathematics Project. As Varga (1965) writes:

> "No one would quarrel with this principle. On the other hand, not everybody would interpret it in the same way, or would make use of it in the same way. Our children appear to know when they have learned something and therefore they know when they are ready for the next task. All the participants feel that there is much hidden potential yet to be tapped. Recent work with children below average ability level convinced us that the type of expansion of mathematics teaching which we have accomplished is applicable in a wide field."

2.8 The importance of individual differences

It is widely known that in a normal class, children of the same chronological age can be at very different "mental ages". Another familiar idea is that one and the same child can be inferior in terms of one ability and yet be superior in another ability. The pity is - as Taylor (1971) pointed out at the XVII International Congress of Applied Psychology - that our schools appreciate less than one-tenth of the approximately 96 different abilities known today. The Project has tried to cover as many abilities that might bear on mathematics as possible, and has let those children who are able to do more in a field produce more in both quantity and quality. We hope that, if we find a wide enough variety of methods, every child can be a high achiever in at least one mathematical field or kind of activity.

The problem of the individual differences is of course, intimately connected with the principle of independence training. With more freedom, individual differences are automatically better looked after. Those children that are able to do more will produce more, in quantity and in quality. Those that produce less are also satisfied since they also have solved problems as well as it was possible for them, having regard to their knowledge and ability.

We do not mean, however, that by educating children to be independent, we automatically look after their individual differences. We merely mean that conditions for solving the problem of individual differences have been improved. For example, we can get to know children much better, observing them working individually or in small groups than by talking to them from the teacher's desk at the head of the class. In organizing this more independent kind of work the teacher has a different task from that of the conventional teacher. He needs to provide the quicker ones as well as the weaker ones with work appropriate to their ability. It must be said that there are still a great number of problems we need to solve in this area of our experience, one of the main problems being classroom organization.

We have already said that children work individually or in small groups. But we work with the whole class, as in the case of games and team competitions. Traditional classroom activities such as mental arithmetic also have their place. Children also invent their own verbal problems, which will fit certain numerical operations that might have been written on the chalkboard. Children can work individually with their working tools as we have already seen, or they can work through the use of workcards on which they can draw or write. Such work can be preceded by verbal discussion about the contents of the workcards, but in the ideal case, there will be no need for this. The instruction cards speak instead of the teacher.

2.9 Internal as opposed to external motivation

This last sentence leads directly to the next principle. The Project wishes to find situations in which every child can find the place where he will enjoy working (or playing - because in this sense it is very hard to discriminate between the two phenomena). As Tamas Varga (1965) expresses it: "Every child is curious. He wants to seek out the secrets of the world in which he lives. Every child likes to learn. This is his nature, just as he likes to eat while he is healthy. While a child receives suitable food, he will remain healthy and will retain a healthy attitude towards eating. Many parents ruin their children's eating habits by being overanxious. In the same way many teachers ruin their children's learning habits by being overanxious."

Fortunately for our physical health, eating problems are less frequent than learning problems. In school we succeed in making learning so unpleasant as to put most children off it altogether, sometimes for life. We regard this to be the rule and we assume that a child does not want to learn so that teachers must, by rewards or promises or by force, induce children to learn and acquire the knowledge which is deemed necessary for him to have. A great deal of energy is consumed in this conflict between teachers and child; in fact the conflict has become institutionalized, and it has been termed "keeping discipline." It is kept in being by the present system of extrinsic motivation, based on grading or marking.

So prevalent is this extrinsic method of inducing children to learn that the majority of teachers would not believe that any other method is possible. A hundred years ago many school masters would have found it difficult to believe that there could exist schools without corporal punishment. This is now admittedly possible and thought even desirable by the majority of people in certain countries. It has taken a hundred years or so to make this step.

Similarly, the process of replacing extrinsic by intrinsic motivation is going to be slow. It would be foolish to discontinue extrinsic motivation at once without any transition, but it is possible to begin a slow shift of emphasis from extrinsic to intrinsic motivation. We aim at a situation in which the pupil considers more important what he learns than what his teachers or his peers thing about him for learning. This means quite simply that we try, as far as possible, to build on the pupil's interest and not on his self-concept. On the other hand, all this does not mean a devaluation of such virtues as duty and discipline. To come back to the comparison with eating: To eat with a good appetite is not contrary to regular and disciplined eating. In fact, the two go together. Likewise discipline and regularity in learning are not contradictory to learning for the love of it.

One consequence of the application of the principle of intrinsic motivation is that it becomes undesirable to allow the weaker pupils to feel that they are stupid and likewise undesirable for the better ones to feel that they are very bright. The emphasis ought not to be on their own persons but on their mathematics, on the job in hand. Children can get used to checking and correcting their own work. Something is correct not because the teacher has said so but because it has been checked and found correct. Of course the teacher should still look at the work and say whether it is correct or not. But there should never be any moral edge to such a judgement. The greatest intrinsic reward both for teacher and child is when both get on happily with the topic in hand. Nobody should be ashamed of having committed a mathematical error. To have committed an error gives an opportunity for discussion and should never be used by a teacher for making a child look small.

Of course whether a learning process is internally or externally motivated depends not only on the tools which are involved but probably much more on how they are actually used. The mechanical use of rods and blocks as simple calculating machines - which is what they may well become in the hands of unimaginative or unwilling teachers - would certainly require the use of external motivation. As Biggs (1962) writes: "If the child does not sense that there is a problem 'there', and that it is a matter of cognitive urgency that he go ahead and solve it, there is little possibility of the flash of insight occurring, and the process would thus appear to be externally rather than internally motivated. On the other hand, where the child does 'see' the connection between symbolic and concrete structures, we are probably justified in assuming the process to be internally motivated (p. 161)."

The OPI Mathematics project tries to help the teacher create an environment for the children in which they can alternate between concrete and abstract work according to their needs, thus remaining in contact with the problem and yet maintaining a high level of interest in it.

It is probably quite clear to the reader that the principles of the OPI Mathematics Project are very interconnected. For example, internal motivation is impossible without taking individual differences into account, winning the teachers over to the cause, using the right concrete material for the abstraction process, and so on. This is the reason why the project has its own "face" despite its eclecticism.

2.10 The Taxonomy of the OPI

In the light of what we wrote in Section 2.1 the reader will easily realize that the aims described in Section 2.4 must become operationalized. Only when this is done will the teachers be helped in their everyday work and the evaluators placed in a position where they can look for the right measurements and standards.

Recently those responsible for the Project made a serious effort to overcome the many difficulties involved. By way of a start they summarised their aims in the cognitive field in a more structured way (See Table 2-1). We hope that this important first step will be followed by a corresponding summary of aims in the affective field, written by the leaders of the Project. This could then form a firm foundation for the development of a system of behavioral objectives.

(Insert Table 2-1 about here)

TABLE 2-1

The Taxonomy of the OPI Mathematics Project in the Cognitive Field.*

	ASSESSING, DEFENDING OR REFUTING IF	CONSTRUCTING
PRODUCTIVE	1. A statement makes sense 2. A statement is true 3. A problem is clearly defined and data are sufficient 4. A problem contains redundant or contradictory data 5. A symbol, definition or a suggested way of solution is suitable 6. A piece of reasoning is correct 7. A solution satisfies given conditions 8. A solution is reasonable or practical	1. Formulating a problem 2. Guessing 3. Planning a solution, finding the tools 4. Finding objects, concrete or mathematical, satisfying given conditions 5. Finding all objects satisfying given conditions 6. Forming a definition 7. Developing a proof 8. Generalizing, extending by analogy, etc.
	UNDERSTANDING	KNOWING, DOING, USING
RECEPTIVE - REPRODUCTIVE	1. Seeing and distinguishing relationships 2. Understanding statements 3. Following reasonings 4. Re-wording and translating between words, symbols, graphs, etc.	1. Terminology, symbols 2. Mathematical statements 3. Routine problems, algorithms 4. Manual skills
	ANALYSIS	SYNTHESIS

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* personal communication from T. Varga

2.11 Curriculum*

The organization of the pupil's work seems to be the sphere most urgently calling for radical changes. Yet it would be wrong to be satisfied with this and to regard the questions of curriculum as of minor importance. Curricula and methodological problems cannot be separated. The decisive role of the new experiments in the establishment of a uniform picture of mathematics has been pointed out. To display this picture is the primary aim of more recent endeavours. It has also been pointed out that even the most up-to-date curriculum implemented by inadequate methods may lead to a fatal formalism. The effect of this interrelationship may predominate in many other aspects as well. The traditional curriculum is always attached strongly to traditional teaching techniques and teaching practice.

Since the problem of curriculum construction has been the subject of discussion all over the world we attempt to outline here some of the key points, paying special attention to those which are not generally treated in traditional curricula and teaching; we show why their omission amounts to a serious misplacement of emphasis. Most of the topics in our list will be found in any up-to-date curriculum, and some of the more advanced concepts have been introduced into experimental curricula and pilot teaching.

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* Suranyi et al. 1972.

Fundamental to any secondary curriculum are the real numbers. To give just one illustration: The elements of mathematical analysis can only be built on an understanding of their properties, yet these properties often remain concealed from the pupils' understanding. The gradual development of clear number concepts must thus become an urgent priority. Besides natural numbers, negative numbers and fractions may appear quite early on, together with operations, inequalities and the like. Rational numbers must form a fully clarified sphere of knowledge for the child in the primary school. Real numbers are concealed in measurement, especially in approximations. If the problems relating to them are not allowed to remain hidden, and the pupils are taught to find their solutions with genuine understanding, then it can be hoped that the concept of a real number will have been adequately clarified for children aged 14-16 years. Relations and functions also start in Grade 1, the latter in the sense of mappings, including geometrical transformations. Geometry today stands in an increasingly close relationship to other branches of mathematics. One has only to think of the linear vector fields. The general problem of the introduction of measure springs from geometrical problems and its complete solution in geometry also is given by the methods of analysis. The problems of continuity form an important meeting point for geometry and analysis. A new branch of science has sprung from the investigation of problems of this nature, namely topology. Elementary topology is a source of many surprising results, and hence also of motivation for mathematics. The integration of geometry into mathematics as a whole by no means contradicts the emphasis of its intrinsic values. The consequent development of space-perception for instance, should be continued. Combinatorics and the theory of *probability* appear at present rather late in the curriculum and little importance is attributed to them. As we see it, combinatorial exercises should have a natural place as early as in the lower grades of the elementary school. The development of the ideas pertaining to probability can also be started very early. The time available and other demands determine where and how much of it may be included in the curriculum. It must be pointed out that a distorted picture of the significance and power of probability calculus is given if its use is restricted to a small number of exercises to be solved by the application of combinatorics.

The role of mathematics in establishing *logical thinking* has been often mentioned. The curricula, however, never go beyond the recognition of this role. It is necessary to point out that the task is to make the pupils conscious of the logical structure of mathematics. This does not necessarily imply the teaching of mathematical logic as one of the branches of mathematics. Whether mathematical logic ought to be taught at all is a matter to be decided upon later, on the basis of the consideration of the given possibilities. One of the most effective principles for organising mathematics into a unity is provided by the study of *sets*. This must be gradually developed from the earliest stages and in such a way that the *organising power* of sets and their relations can be exploited throughout all other branches and stages of the subject. Another principle of organization is offered by the concept of *algebraic structures*. It is quite obvious that the introduction of certain algebraic structures and their more detailed study will be included in any up-to-date curriculum. At present it would be hard to say how and to what extent they will be included. In more recent applications, mathematics resting on completely elementary principles has played an important role. Most of this could be taught in schools but would take us far beyond the scope of today's curriculum. To decide what and how much should be taken from these application into a new curriculum will be a task for the future. It must be emphasized that the most important point is to give the pupils a clear appreciation of the basic ideas and to provide them with a fully understood knowledge that can be easily increased. In keeping with these demands the pupils ought not to be taught more than they are able to absorb.

This outline can be regarded as the first attempt to categorize the problems. Even if it is acceptable today it may require alteration after a short time. However, it will by then have served as a starting point to solve everyday problems. Accepting tentative amendments as though they were final or rejecting them altogether because of their shortcomings would be destructive of our fundamental expectation, that more viable and therefore more lasting modern curricula will be developed through a long succession of trials, errors and improvements.

2.12 Teaching aids (facts and desires)*

The situations to be analysed mathematically must, especially in the beginning, be set up in concrete materials, which pupils can see, touch and re-arrange. A large variety of such materials is now available on the educational market; many of them are flexible enough in their design to lend themselves to use in a large variety of mathematical situations. Some of these materials have become classics through their versatility and simplicity, for example, the coloured rods of Cuisenaire, the multibase arithmetic blocks and the logic or attribute blocks of Dienes, the nailboard (Fig. 2-1) or geoboard of Gattegno. These can and must be complemented by others if through richer experiences we wish to ensure that the child is readily led to abstraction. It might also be useful if the teacher designed materials of his own to promote his teaching, possibly with the pupils themselves playing a role in this work.

(Insert Fig. 2-1 here)

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* Suranyi et al. 1972.

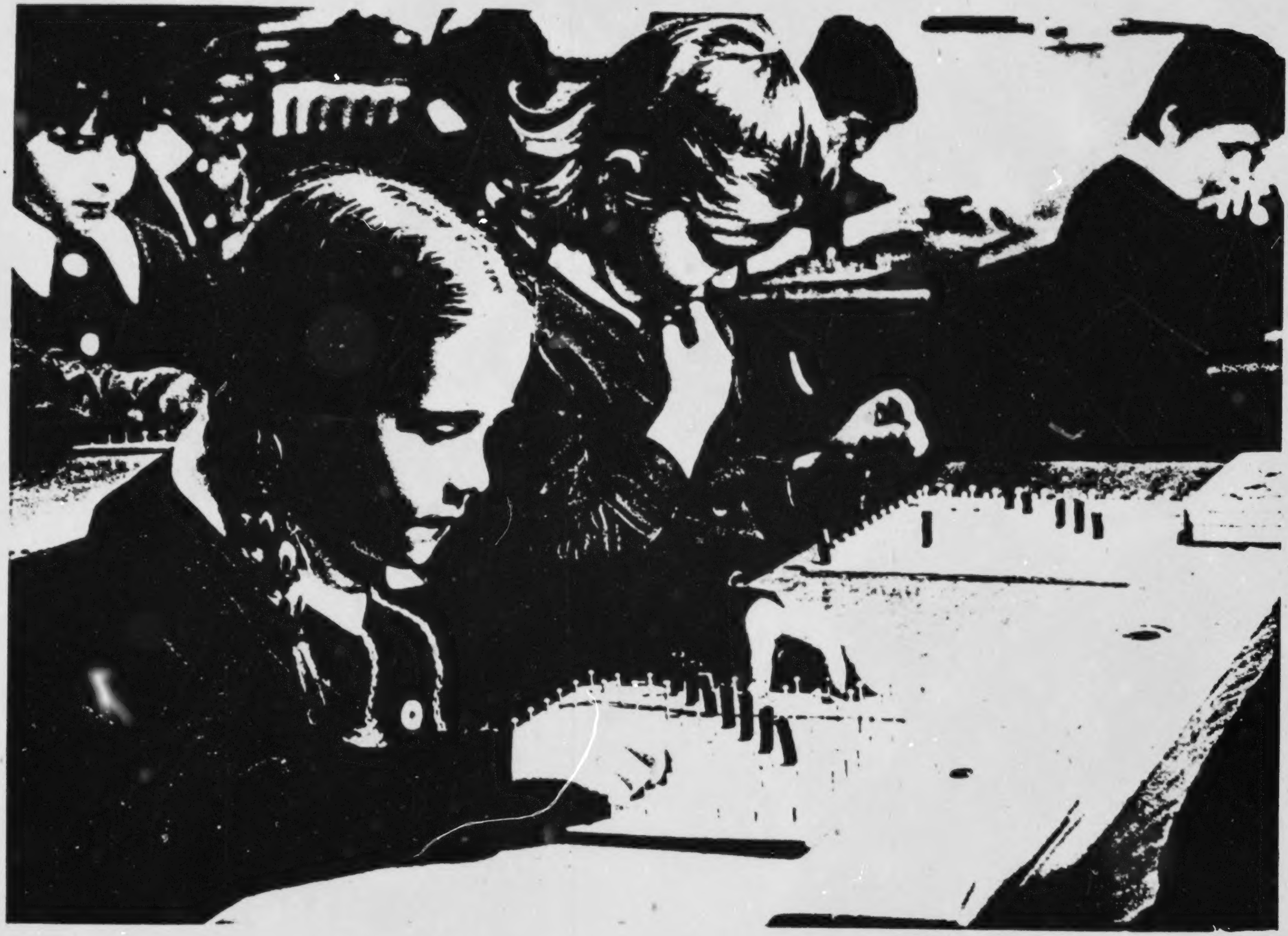

Fig. 2 - 1 Working with the Nailboard.

Mathematical experience centering around manipulative activity is of primary importance, especially at the lower levels. Drawings and written symbols become more important as the new knowledge is related to the mathematical concepts already learned, to numbers instead of to quantity, to geometrical figures instead of to objects, and so on. But pupils should remain aware of the rich variety of concrete meaning that relates to mathematical sentences, whether uttered in words or expressed by formulas.

At the level where pupils work with symbols and drawings worksheets have proved very useful. These can be of great variety and serve many purposes. It is advisable to have them available in large numbers, and never to let them become a rigid system of chores. Teachers can design new ones themselves, specially adapted to certain emerging problems, to certain shortcomings, or for helping certain pupils. For the rapid production of such worksheets it is necessary, of course, to have available the necessary labour and machines.

Although experimentation seems to have concentrated on the lower age ranges it seems reasonable to assume that an appropriate supply of aids will promote independent work at any level of learning. If aids assist motivation, understanding, assimilation and retention then they ought to be of use wherever learning is in progress, though of course the particular forms they assume must change as the child grows to maturity.

The problem of the more classical aids of teaching guides such as the textbook, still remains unsolved. At present, the textbook is mainly used by the teacher to provide a supply of exercises. Although more recent textbooks and teachers' manuals have improved the situation, the disregard of textbooks is still a little worrying. Ways of exploring written information must be probed because it is in book form that the bulk of knowledge is passed on from one generation to the next.

It may be that the structure and style of the textbooks are responsible for their disregard. We have to endeavour to use a style intelligible to our pupils. The textbooks should not be confined to presenting mathematics in its final perfection. They should rather throw light on how ideas evolve; they should give the pupils a lead, explain the most important ideas and their power and encourage the more advanced reader to analyse texts on higher levels of abstraction, precision and conciseness.

With all that granted we still have to reckon with some inevitable intrinsic difficulties. The mathematical textbook is eo ipso more concise than the majority of literary works or books dealing with humanistic disciplines. One must learn even to read literary works in order to become more receptive to what the work has got to say apart from its actual plot. This is much more true for the reading of mathematical books. One of the initial steps can be to use the textbook in the course of the lesson: The pupils can read or listen to one piece and then discuss what it was all about, put questions and finally give an account of the content in their own words.

Prominent also among the new methods and aids is programmed learning and the use of teaching machines. The use of programmed books has spread both in school teaching and in individual learning. The majority of the above mentioned worksheets can be regarded as programmed aids; at least it can be maintained that they bear a close relationship to them. The applicability, realization and limitations of these methods are again subjects for much further investigation.

Even prior to such research, some points must be considered. Programmed materials take many forms. The tendency of imparting knowledge to the reader mostly through correct answers is in direct opposition to the previously outlined idea of learning through mistakes. The experience gained from the recognition of the errors plays an enormous role in mathematics learning. The application of the multiple-choice system in the teaching of mathematics is limited,among other things, by the fact that it discloses the nature of the answer to be sought, the identification of which very often plays an important part in the solution. Another drawback is the possibility of selecting the correct answer without actually solving the exercise. The danger of the programmed materials lies in their tendency to set the pupils on a road leading to one given sequence of thought, to give them only one possible way for solution, at the same time limiting their independence and their initiative. Nevertheless, they can lighten the burdens of routine and repetition for the teacher and provide an orderly introduction to a particular part of the subject which the pupil may have missed through personal absence.

The role played by variety has been emphasized many times in this book. The complex task of teaching can only be performed through the application of numerous other methods (Fig. 2-1). Accepting any single method or approach in the hope that it will be the final solution of the problems of teaching mathematics is certainly harmful, especially if the approach is believed to be in line with modern ideas and up-to-date technology. This may be the case with much that is sold as programmed learning. It may become popular through momentary results mainly because it does not require too much fresh knowledge from the person who chooses to apply it. But it can never ensure a harmonious and multilateral development of mathematical skills and knowledge, and despite its partial results it may sidetrack both the teacher and the learner from seeing mathematical truths as they really are.

(Insert Fig. 2-1 here)

The use of tables and other devices such as the slide rule should also be mentioned in this connection, although they do not directly provide mathematical information. The majority of the pupils will need to be able to use such devices later on. Their regular use in class is therefore important, to say nothing of the fact that they motivate the pupil by enabling to achieve reasonable numerical accuracy at high speed. Where conditions permit it should also be possible to introduce older pupils to the power and speed of some electronic methods.

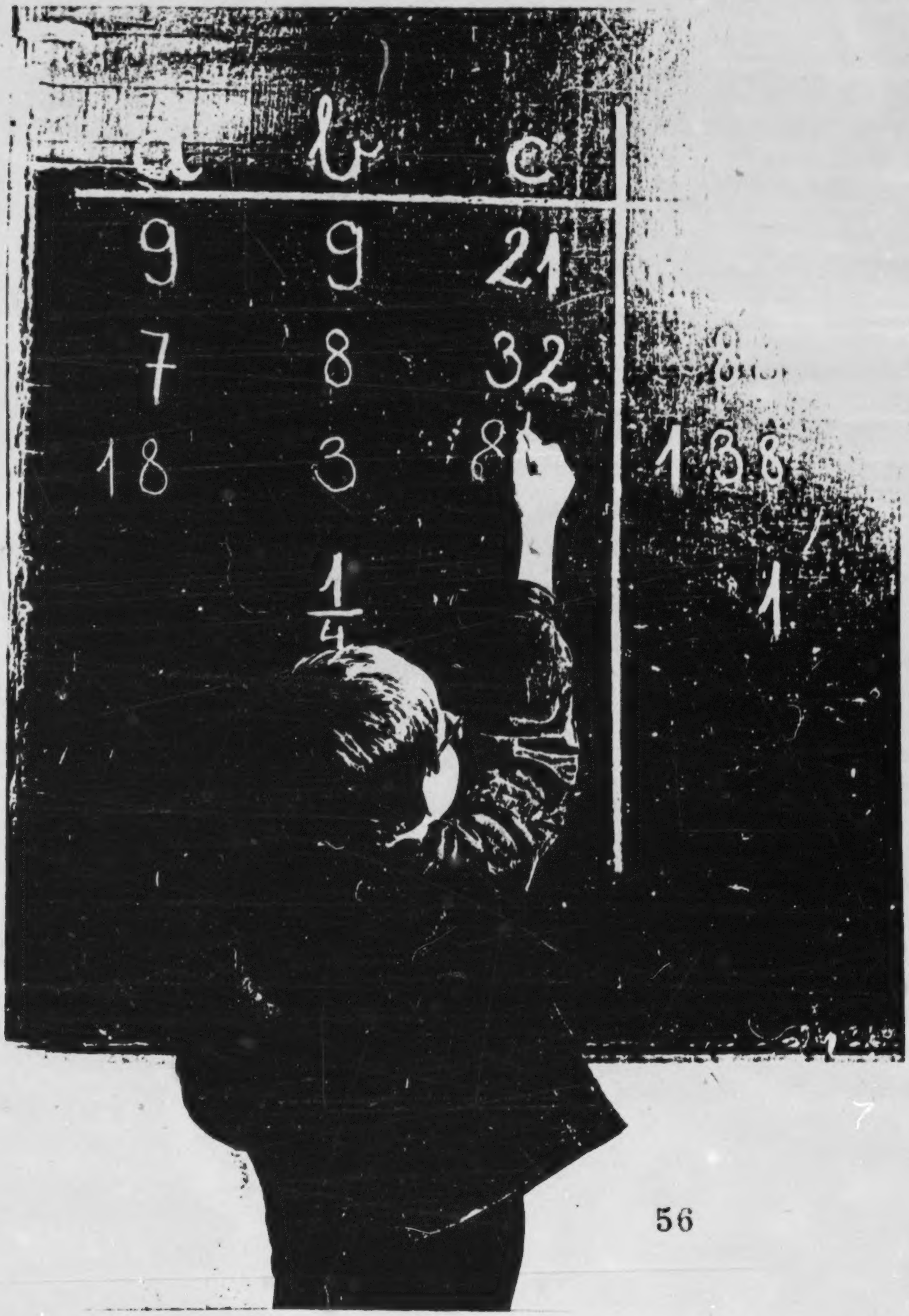

Fig 2 - 2 The "Traditional" Blackboard used for Modern Mathematics

2.13 A Geometry lesson in an OPI Mathematics Project School

A vitally important step in evaluating an educational project is understanding the environment it creates to reach its objectives. There is no room here to describe fully this side of the investigation; the environmental characteristics we considered important are found in Table 3.

To get a feeling of the climate, we include here a description of a lesson given at the very beginning of this project.*

The lesson begins with the teacher sketching the figure below on the board, doing so without saying a word. During this time a pupil is working on a problem at the other end of the board; the problem has been selected by him from amongst others written out on cards.

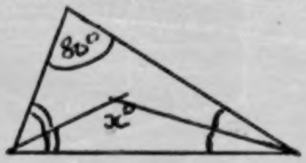

Figure 2-3

After a minute or so several pupils begin to put their hands up. The teacher waits for a few more hands to go up, after which he calls on one of the pupils.

"In the big triangle one of the angles is 80°, so the sum of the other two must be 100°", declares the pupil, "The sum of their halves is 50°, which leaves 130° for the angle x."

The teacher sketches a similar problem, this time with 70°. He says:

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* Written by T. Varga (1963)

"Do this one by yourselves and write down the answer."

He strolls around the pupils' desks, and notices the following figure in one of the note-books, which he proceeds to draw on the board.

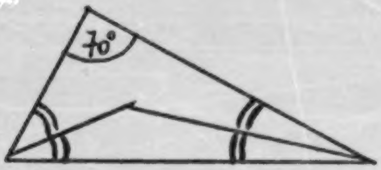


Figure 2-4

T. "What do you think of this figure?"

P. "It isn't right. We don't know whether they are all equal."

T. "If they were equal, how large would each one be?"

There is some general discussion, two different values are proposed; the teacher does not take sides; it is clear that the pupils are quite used to this; they do not expect the teacher to adjudicate on points at issue as some kind of oracle; they are not even looking at him to obtain a hint from his expression. The discussion in which the teacher acts only as a kind of catalyst is soon over and a decision is reached. In the meantime even the slower ones have solved the problem.

A little later again the teacher turns to a child working at the blackboard:

T. "Are you ready? Well, let's hear it."

On the board the figure drawn below can be seen, roughly sketched by the pupil who had been working there.

P. "I had to find a construction for a circle which touches a certain circle at one of its points, and also touches a certain straight line. Two conditions needed to be satisfied. One condition is satisfied by those circles whose centers lie on this line. (Here he points at the straight line which passes through the given point on the circle as well as through the center of the circle.)

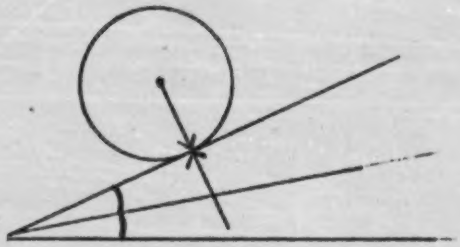


Figure 2-5

"The second condition is not enough in itself, since a lot of circles could touch this straight line. So I added a further condition that the circle to be constructed also had to touch this straight line." (Here he points at the straight line touching the circle at the given point). "The set of centers of those circles which touch this line as well as the given one is the bisector of this angle." (Here he points at the bisector of the angle on the figure).

At this point somebody puts his hand up, but the teacher asks him to wait for a while. The pupil continues:

P. "There is one point of intersection of these two sets of points, which satisfies both conditions." (He points at the two sets of points while he is speaking.)

T. "Any remarks?" (A large number of pupils wish to be heard.)

T. "Claire, what have you to say?"

Claire: "He never finished the construction. He should have drawn a circle whose radius is the distance between the point of intersection and the given point on the circle. The point of intersection is the center of the circle."

T. "Any further remarks?" Several pupils are very excited at this point and demand to be heard. The teacher calls on one of them.

P. "The problem has two solutions. These two straight lines (she runs to the board and points) make more than one angle and there is another bisector. These two bisectors together form the second set of points. So the first set and this set have another point of intersection.

She goes and completes the diagram on the board. But the second intersection would already lie off the board. She is worried.

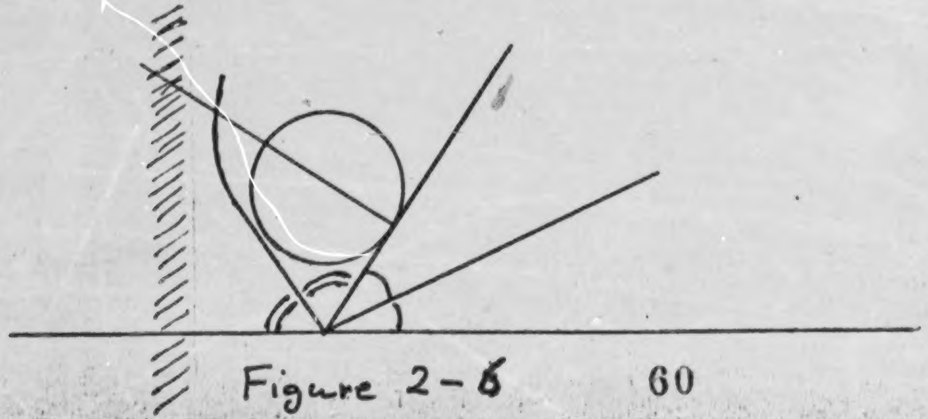


Figure 2-6

The teacher comes to her rescue and says:

T. "Let's cheat a little bit", and draws the figure (shown on previous page). "Any further remarks?"

P. "Now they are not outside each other but inside..... the given circle will be inside the circle to be constructed."

Chapter 3

A Visitor Looks at the Sherbrooke Project

Shortly after his appointment as Director of the Research Centre in Psychomathematics at the University of Sherbrooke in Quebec, Canada, in 1967, Dr. Dienes visited some nearby elementary schools. He found their mathematics teaching traditional in both content and approach. The first of several schools to respond to his suggestions for change was the Ecole Eymard. A new mathematics programme was introduced and allowed to grow up with the children from the start of Kindergarten (age 6) to the end of grade 6 (age 13). When Professor Lant visited the school in the spring of 1973 he found the Dienes Project established throughout the school and recorded his impressions.

3.1 Impressions on Entering

The classroom door opens and after a 'good morning' from the teacher and children which sounds unmistakably genuine one is invited to walk round, hold conversations, or handle any equipment that has caught one's fancy. Visitors are clearly part of the game. One has not disturbed the peace by one's appearance. After a few minutes all is back to normal and the newcomer is left free to discover what is going on.

There are no tidy rows and columns of desks, just tables, little chairs and chests of drawers scattered about the room. Here is a little group of youngsters engaging in conversation that sounds very exciting. Over there is a little fellow checking something on the blackboard; he will return to his place in good time, perhaps stopping for a short chat on the way. Two little girls are coming up; they wish to pass to the far corner of the room where the cards and blocks for their next assignment are hidden in the fourth drawer down. The air is busy but nowhere does the volume of noise need cutting down (see figure 3-1).

(Insert Figure 3-1 here)

What has happened to the teacher? A moment ago he rallied the class for the welcome but now he seems to have let business run its way. Ah, there he is, crouching down over some plastic figures and pointing out their arrangement to a little group. A few minutes later he will have moved to give a few directions somewhere else.

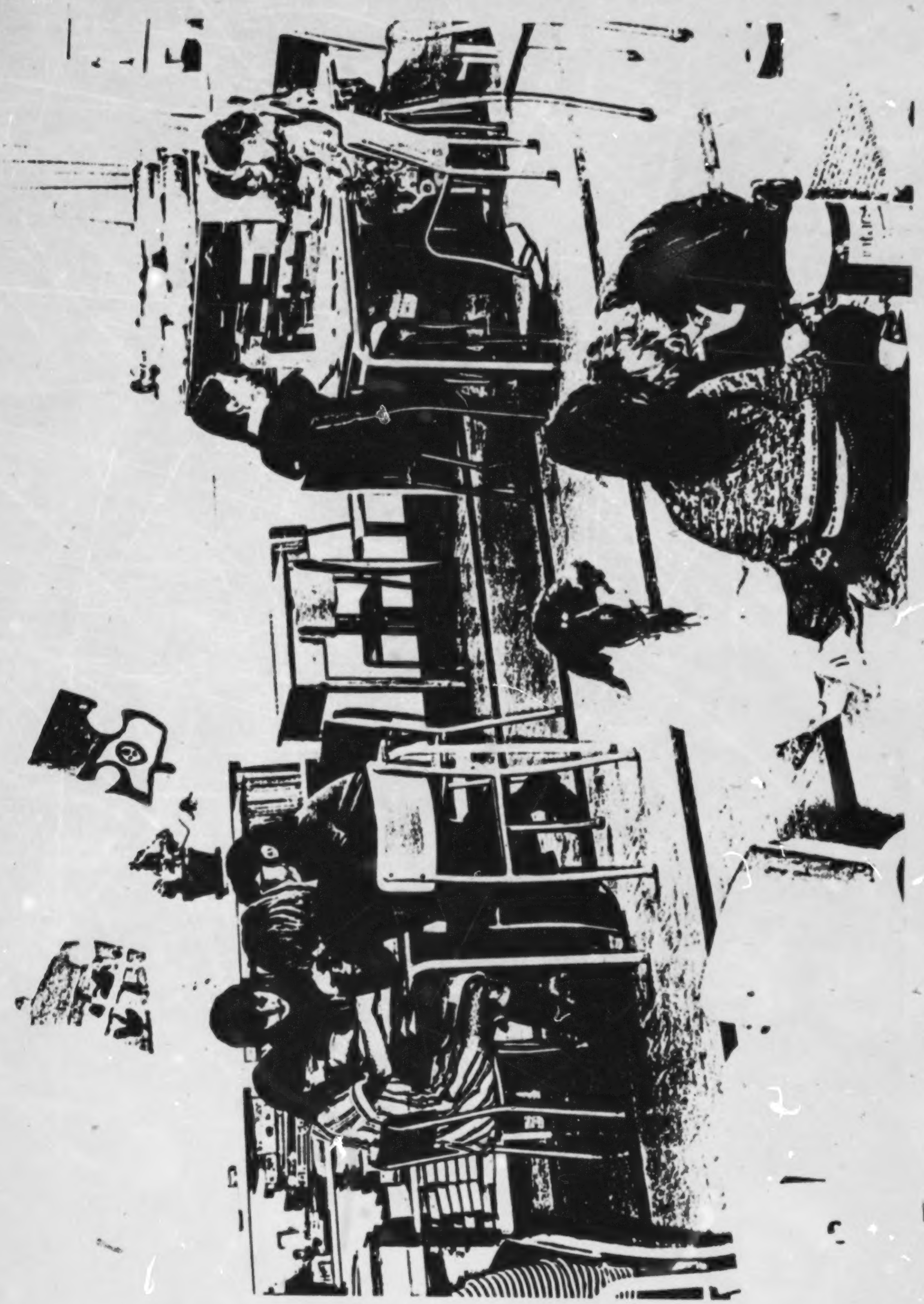


Fig. 3 - 1 There are no tidy rows or columns of desks

To the visitor whose employment is not in education all this may come as a shock. He will reflect on his childhood, when all worked on the same page of the textbook from 9 to 10, did only what they were told and quietly sighed with relief at the striking of the clock. In the Dienesian mathematics room the situation has changed. The overall purpose is to promote mental growth through the understanding of mathematics, and this seems to be a less simple matter than is commonly supposed. An hour's daily drill in rows and columns looks very tidy. The written work is easy to cross and tick, but the children are gradually dulled into a kind of automatic behaviour, totally divorced from the high-pitched thinking that has made mathematics the subject it is.

Should the visitor happen to be acquainted with today's trends in education then all this will seem far less heretical. Now and then the television informs the public of the silent revolution which hopes to build better and better schools for tomorrow. Psychologists and the social conscience are at last approving of what Pestalozzi, Froebel and Montessori preached some time ago: The environment of the classroom must be that which accelerates growth by providing for the child's needs at each stage of his development. Curiosity, movement and communication are necessary ingredients in successful learning, no matter what it is that is being learnt. And for this reason they are also fostered here.

3.2 A first look round.

Recovering from his initial surprise the visitor will most likely begin to look for the mathematics in which the children are engaged. Here a second shock lies in store. School mathematics is not what it was 20 or even 15 years ago. Back through our century learned societies often recommended changes in the kind of mathematics that was taught, but their voices rarely reached the elementary teacher before the Second World War. Following that event inventions in industry snowballed until it became clear that many employees would have to know more and more mathematics; the calculations composed of the basic rules of school arithmetic, it was argued, formed a vanishingly small part of what there was to know, and in any case proficiency in them was a less crucial matter with the coming of the computer.

On his departure the visitor will be convinced that arithmetic also forms a vanishingly small part of what young children can do. But for the moment he must trust his eyes when he sees charts, cards, blocks, tracks, arrows, and plastic men lying about on the work tables or the floor. The children are arranging, sketching, recording, disputing and agreeing, yet nowhere is there a single calculation with numbers to be seen. Mathematics without number, and yet mathematics in bewildering profusion! The only immediate indication that all might be well seems to come from the self-confident poise of those who are doing it all, and doing it as though they had never known anything different. And indeed it has all been set up to good purpose, as even a cursory glance at Dr. Dienes' writings will show. Viewed in the most general way mathematics is the study of relations between objects. What the objects and relations are does not matter so long as both can be described without any margin of uncertainty. The relation between a man and his dining club is clearly that of membership; we can always decide whether Mr. Jones does or does not belong to a particular club. On the other hand there is the relation between a man's life and the stars, as studied by astrology. Our knowledge of it is vague and of little use for starting any mathematics.

Our everyday environment contains many objects between which clearly defined relations exist: Looking at a row of trees, for example, we become aware of relations such as 'larger than', 'further than', or 'to the left of'. All of these play an important part in geometry and if we look in the right direction nature is prepared to present them to us in a simple way. More often on the other hand it is we who impose the relations on part of the environment we have selected: In chess or checkers every piece stands in a definite relationship to every other piece of the game. Rules in which these relations are clearly laid down form the beginning of a mathematical system, though in fact systems which usually bear this name differ from the games in the complexity of their rules, also in that the 'objects' which are manipulated are not nicely carved pieces but symbols like $\log_e 2$ or $f(x)dx$ written on paper.

Dr. Dienes has exploited this transition from the concrete to the abstract, from the simple to the complex, in an extensive programme designed to lead the children into the grasp of serious mathematics in a natural order. Most young children are far from abstractions and complexities, but already the first visit shows that their enthusiasm for detecting a multitude of relations among the cards, blocks, tracks and other equipment can be aroused and maintained.

Here lies a collection of cards. Each card has drawn on it a house with windows and chimneys. Some houses are blue, some red and some green. One house of each colour has 1 chimney and 3 windows, another 3 chimneys and 2 windows, yet a third 1 chimney and 1 window, and so on. Over there lie 4 kinds of geometric figures: Triangles, squares, rectangles, and circles. Each figure is moulded in [illegible] different thicknesses, 2 different sizes and in the 3 colours of red, blue, and yellow. Elsewhere again stands a table covered with a heap of plastic rectangles. These are embossed with tracks fitting together like the tracks of a model railway. At certain points holes appear into which stoppers can be inserted to act as signals for objects possessing certain properties to pass or not pass along the route.

Children who see the equipment for the first time view and feel it with excitment, like the contents of the toy box they find under the Christmas tree. Far from being regarded as a waste of time this initial appraisal is treated as a first stage involvement on which more purposeful activities can later be built. Just how much of this later development the visitor will be able to see depends on the grade of class to which he is first introduced, on the length of his stay, and clearly also on his rate of comprehending what is going on. The sooner he can supplement his direct impressions with some explanations the less fragmented and prone to misinterpretation will be the picture he takes home.

3.3 The children's personal relations

It may require several visits to the classroom before one can grasp the patterns in the children's activities. But whatever they are learning or not learning the children are persons, formed from one day to the next by the relationship they have with their classmates and the teacher. (See Figures 3-2,3-3,3-4).

Fig. 3 - 2 Investigating tessellations and symmetry

Fig. 3 - 3 Preparing a tree diagram

Fig. 3 - 4 Mutual discussion

(Insert Figures 3-2,3-3 and 3-4 here)

Of this relationship the visitor soon becomes aware. Excepting the start and finish of an activity period or the occasional short demonstrations by the teacher no rigid discipline is imposed on movement, pace or speech. The children play and talk their way into mathematics, not away from it. If John moves over to his girl friend's table to look at what she is doing nothing will be said. He has taken a few minutes rest; before long he will be back in his group, busily arranging the cards with their houses and chimneys. Looking out of the window is Susan. Should she be recalled from a day-dream or will she come round herself? She seems very still, then suddenly walks to the blackboard with determination and alters a line or two in her sketch. She must have been weighing possibilities and suddenly made up her mind about what to do next. Maybe her friend Anne on the next table will reach the same decision more quickly when she takes her turn. It does not matter. The child's time is precious for what it does to him, not for what precisely he does in it. Accustomed to their liberty these children are far removed from "Oh, here's another lot of sums. Let's get down to the, through them and hope for the best!" To them mathematics is creation in which their whole being is involved. It may be creation of rules and creation within rules but because it is creation it brings with it contentment, perhaps a kind of aesthetic satisfaction which the research mathematician claims to experience in common with the painter, sculptor or musician.

The children are proud of what they are producing and ready to share their suggestions with each other and the teacher. If the visitor stands close by a table and looks as though he wanted to talk they will explain to him what they have discovered and what they are still trying to do. Suddenly a group in the middle of the room starts to buzz more loudly than the rest. It is not mutiny but high-pitched excitement over some unusual brain-teaser. No one punches his neighbour for being a fool or claims a monopoly in inspiration. Soon it all dies down because part or all of the answer is out, a joint production from which each worker has learnt his share. The better ones seem to teach the slower ones by example and the constant exchange of information. All have to practise clothing their thoughts in language until they are readily understood.

As the bell rings and the children line up to go out for their play the visitor will feel tempted to walk up to the teacher and after some congratulatory remarks ask him how he is able to keep track of his objectives with 30 children pulling here, there and everywhere. "I go to the groups one by one and when I think that a particular group has understood some part of the programme I sit down and give this group an evaluation. The rest carry on as usual." The answer is disarmingly simple. Until the crucial stage of formal evaluation is reached at the end of the scholastic year the terror of the mathematics test on Friday morning for which one crams the night before is gone; and when the year really draws to its close the children have enough knowledge and confidence behind them to put up a good show.

3.4 Other subjects are also there

The visitor has stayed for the children's return after 15 minutes. Walking round by himself he notices a table on which lie what appear to be children's drawings. Quite near to this stands another table covered with sheets of printed squares. There would be nothing unusual about this were it not that along the sides of these squares are printed words from the vernacular, ordinary words like 'sweet', 'light', 'bitter' and 'dry'.

Is this mathematics as well? If not, then what is the material doing on the tables during a mathematics lesson? Later he will ask for clarification but right now the workers are returning. Without fuss or regret about another spell of confinement they take up their places and continue. The group at the table with the squares cut them out and then match them up again in such a way that where a word and its antonym are given they come to rest against each other along adjacent sides. This is classification and grouping as in the case of the cards and plastic shapes, except that here it has a dual purpose: The objects are words whose form and meaning make up the content of an English lesson as well.

It is all a little puzzling, and rightly so, if one considers the finer and finer specialization towards which occupations in civilized society are advancing. But then the society here is an embryonic one, where different priorities have been selected with profit. The claims of specialization are outweighed by considerations such as change in occupation, cost in the duplication of equipment, and the stress on co-operation and creation as underlying the gain of every form of knowledge. If the order to march in step has disappeared from the mathematics lesson why should it remain in force for the time-table as a whole? Art, language, geography and history can and do exist side by side with mathematics in the same room. Nobody seems to worry about unfair shares, about his subject catching some infection from shapes, arrows or hysterious codes.

3.5 Numbers too have survived

On his early walk round the classroom the visitor will as likely as not have been shocked by what the children did in the name of mathematics. Of course number somehow lurks in the background wherever people are busy with a multitude of objects, but not a single child seemed involved in giving number as such its due attention by working at sums.

Another teacher enters and invites him to look in next door. All seems much the same until he gets to a table where the children are handling some brown blocks and sticks of different sizes. The glamour of the lovely colours is missing but this is too superficial to be of serious interest. The visitor walks closer and notices that one boy has written down a row of figures and is just finishing the next one underneath. Arithmetic must still be alive!

How indeed could it be otherwise? New branches of mathematics it is true, are growing from year to year at an increasing rate; despite this an old and difficult branch, the theory of number, continues to be held in great esteem. Again, science and business call for longer and longer calculations to be done at greater and greater speeds, with the result that some mathematicians are leaving everything else to search for efficient ways of presenting such calculations to the electronic computer.

Far from ejecting number work from the classroom Dr. Dienes actually gave it close attention in his pioneering days some fifteen years ago. He produced wooden blocks to help children grasp the way in which we present our numbers by placing the numerals 0, 1, 2 .... in appropriate positions. In his equipment the number twenty-three, for example, is represented by three little cubes and two sticks known as LONGS. The latter have the same width and height as the little cubes but are ten times longer (See Appendix 2).

Ten of the LONGS then make a FLAT which is both ten times longer and ten times wider than a small cube. Finally there is a fourth block to represent one thousand. This is ten times longer, ten times wi[illegible], and also ten times as high.

Organizing and understanding the representation of numbers is a habit which many children do not seem to acquire with ease. To sift the essential from the unessential Dr. Dienes went on to design collections of blocks based on organizations other than ten, namely on twos, threes, fours.. .. . .. right up to twelves. These provide great scope for activity and help children to surmount hurdles such as carrying. Paul, at whom the visitor is looking, has finished his addition sum as surely as he would have done in a classroom 30 years ago. But he seems to have worked with more self-assurance. On being questioned he reveals a good understanding of why he has carried, how much he has carried and where exactly he has written the number that had to be carried. The intricate convention about the position of the units, tens and hundreds has not been put before him in the ready-made state which man's thinking took millenia to evolve. Instead the blocks help him to retrace the invention in its growth; and the practice with different systems of blocks sharpens his awareness that position and consistency are more important than the choice of the number 10.

3.6 Dr. Dienes in the classroom

During one of his visits abroad a newspaper appearing just after another moon probe had left mother earth acclaimed Dr. Dienes as the 'father' of mathematics teaching. If the visitor is fortunate enough he will see him enter and take a look round in person. When this happens his standing as an innovator and theorist is forgotten; he just becomes the children's friend, a friend who has the knack of painlessly pushing their mathematical insight a little further ahead.

The heavy administrative commitments which match the extent of his travels and researches have never allowed Dr. Dienes to forget that the central reality in the whole project is the child, with his ayes and nays to what is tried on him. In the last resort teaching is a personal matter and the inspiration for striking out in a new direction derives from first hand experience more than from volumes of reports and statistics (See Fig. 3-5).

(Insert Figure 3-5 here)

Dr. Dienes chats about, around and away from mathematics. The children enjoy the rapport, show off with the results of their mini-research and fall back on their regular occupation when he moves to another table. Eventually it is the teacher's turn for a tete-a-tete. He knows that his queries will get a patient hearing and be resolved. This done Dr. Dienes knows that now, if ever, he must sell the teacher another line, new sketches on a different number of cards, or just a sheet of instructions to try a subtle variation with material that is already known. If at this point the teacher reveals that he himself has tried something new so much the better. The spirit of meaningful co-operation extends to all alike. The worksheets handed to the children are liberally filled with "Try a different or similar combination yourself!", and the teacher needs no special invitation to try his hand at creation as well.

Fig. 3 - 5 Dr. Dienes in the classroom with five year olds

To day time and distance prevent many children working in his projects from seeing the 'father' in person. But those living near his home may even find themselves invited to supplementary schooling on a Sunday afternoon. They do not decline since they experience work and play as close partners at school. If the visitor has been asked for tea he may not find his host until it occurs to him to look in all directions on the floor. There he is at last, stretched out by the crouching child and waiting until each point of a finite geometry has been found its rightful place.

3.7 The preparation of the teacher

During and after tea there will be much for the visitor to ask and even more for which he will have no time to ask. How easy is it to find capable teachers willing to change over to the new approach which looks so demanding in its organization if in nothing else? This is part of a wider problem with which those responsible for the training of the elementary teacher are faced. On the one hand it is true that he can lead children into knowledge the more he knows about this or that subject himself; on the other there are strong reasons for letting him take the same class in a large number of subjects. As a rule he receives a broadly-based training which together with self-study or further courses during employment has to suffice. There is a growing tendency to lengthen his initial training and encourage him to advance to the standard of at least a general degree in 2 or 3 subjects. Despite these improvements a mathematics specialist in the elementary school remains a precious exception.

Every co-worker in the Dienesian project must somehow acquire a thorough grasp of its principles. There are no make-shift rules on which he can get by, no artifices for hiding his lack of knowledge. After making much additional effort himself he must accept it that the children are likely to bring him questions to which he has no answer to hand.

Although few regular courses exist at universities or teacher training institutions Dr. Dienes has, through regular workshops and consultations in the classroom, mustered sufficient teachers with the right personal and intellectual qualities to conduct classes in countries as far apart as Hungary, Canada and New Guinea. It is difficult to conceive how anyone not wholly in sympathy with the approach could induce children to receive its message. The thoughts of adults are more complex and more surely established than those of the young. For this reason they constitute a conscious or unconscious resistance to innovation, especially if this innovation is fraught with the hazards of humiliation. The children in their Kindergarten are starting with a clean slate, but the teacher has more likely than not been at the learner's end of traditional mathematics for some years. In one sense he must be untaught and start again; in another he must make himself director of a sizeable project and acquire such mastery over its details that he can admit any visitor with ease. When somewhere in the distant future the first teacher appears who has passed through the system himself we can expect its adoption to spread at a faster rate.

3.8 Taking a closer look

The first day's visit and discussions are over. There are many clear and good impressions but the visitor feels swamped by a mass of details which he can not properly connect. If he can spare a further 2 or 3 days he must start a more intensive tour in a Kindergarten or Grade I class, little by little making his way to higher levels from there. Even this may not help him over his confusion unless he settles on a definite item of instructional equipment and confines himself to tracing the uses to which this can be put.

Attention has here been focused on the equipment known as trimath ('tri' as in tri-angle). This consists of plastic triangles in the six colours of red, green, yellow, black, white and blue. Each triangle appears with [illegible] 1 and 2 holes, also with 0, 1 and 2 vertices cut away. Generally it is immaterial whether the children' think of the shapes as triangles with vertices removed, or as polygons whose vertices number 3, 4 and 5. In the interests of simplicity they refer to them as triangles, hats and kites. There are 54 pieces in all (3 choices of vertex, 3 choices of hole and 6 choices of colour), but in many investigations only a small number of them is used. Considerations of space have made it advisable to change the colours to 3 in this account, namely white (w), grey (g) and black(b). This leaves 27 pieces in all.

(Figures 3-6, 3-7, here)

Fig. 3 - 6 Studying reflection

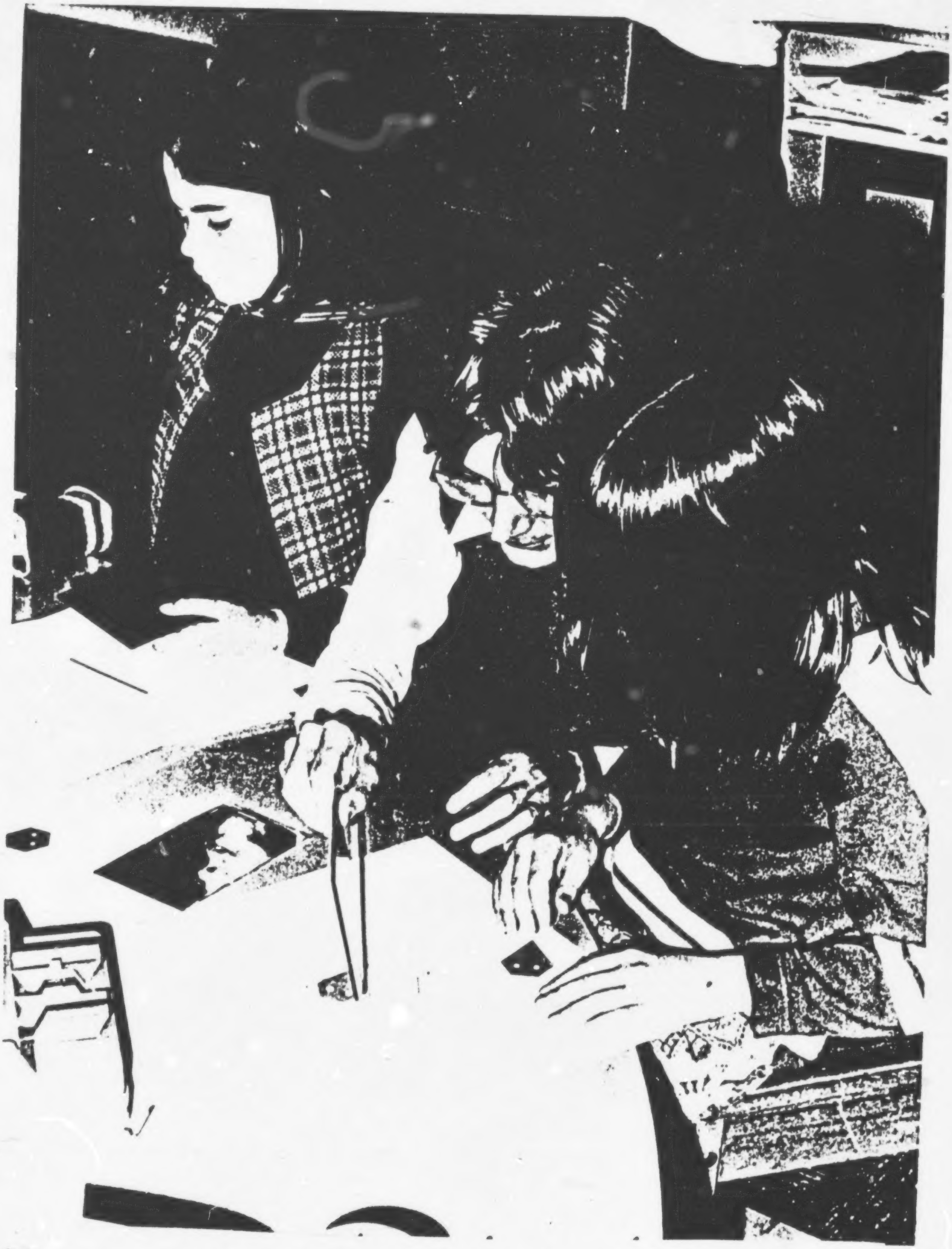


Fig.3 - 7 Children working with Trimath kit and mirrors

(i) A situation of free play

Already during the first day's visit it was noticed that children will tend to look at a piece of new equipment and begin to handle it rather like a Christmas toy. This kind of situation the visitor will most frequently encounter in the lower classes of the school. It is the one time that there is nothing for the teacher to prepare, but much to observe. Little Francis just delights in spreading the pieces out haphazardly, but next to him sits Susan, six months younger but more ambitious in her dealings with the triangles, hats and kites. Will they fit against each other until they completely cover all of the table top? It is worth trying, perhaps with the triangles alone, and sharpens her discrimination in any case.

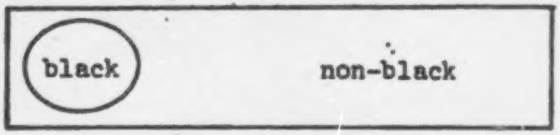
(ii) A game of familiarization

The children, it was noted, learn from working with each other. On this table they already have a pretty good idea of the kinds of pieces that the toy box contained. Taking the 9 white ones they appoint one of the group to do a little hiding while the others turn their heads away. When finished he challenges them to name the piece he has put away. The first one to shout "the kite with two holes" wins the round. There is a little scramble and then it starts all over again. Winners and losers have sharpened their sense for handling words.

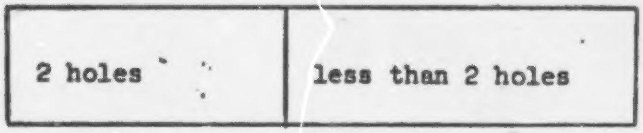
(iii) Starting some classification

In every science observation is followed by classification, the first attempt to impose our intelligence on the things with which we have to deal. Young children are no exception and the visitor pursuing the trimath will see pieces of white cardboard with charts like these:

(A)



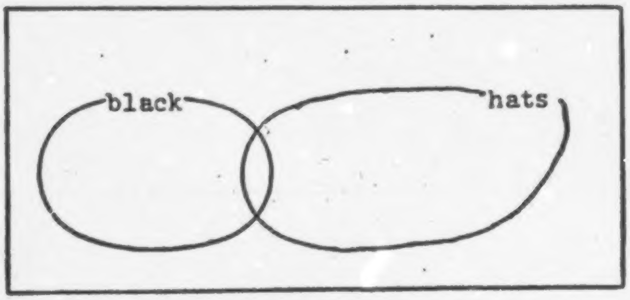
(B)



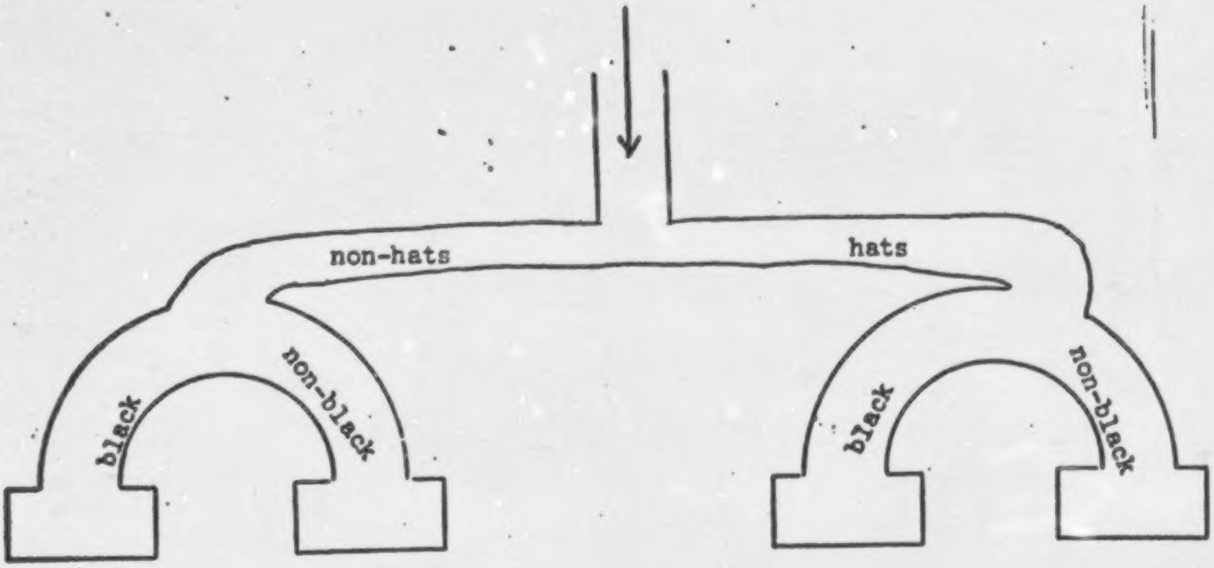
The children focus on just one property and then place the pieces in the appropriate region of the chart. If the full entire set is used there will be the colours grey and white to come under the heading of 'non-black', also both 0 and 1 as the number of holes less than 2. Should this prove too difficult then pieces with one of the properties are removed until more confidence is gained. For example, with the grey pieces removed then as far as task (A) is concerned the words 'white' and 'non-black' have identical meanings. The children may later check that this is so by interchanging the two words in instructions given to each other.

More difficult are the following:

(C)



(D)



(C) provides a first taste of the Venn diagram so commonly seen in school texts today, and (D) represents the same classification as (C) in what is often referred to as a decision tree diagram. To those children who have forged ahead and are waiting with everything neatly arranged the teacher will suddenly put some searching questions:

(1) Would the four different collections into which the pieces have been sorted remain the same if 'hats' and 'black' were interchanged in either diagram?

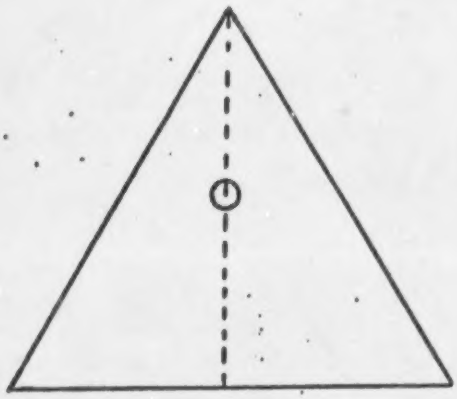
(2) Is there a change in diagram (C) corresponding to every change you can make in diagram (D), and vice-versa?

The second question is designed to test the child's power of abstraction, to what extent he has been able to free his mind from the particular shape of the diagram and grasped the essentials of the situation.

(iv) Trying some geometry

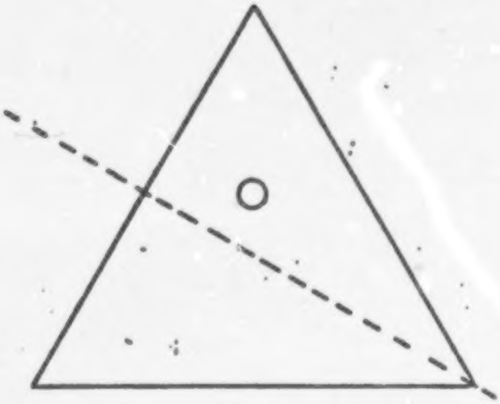
Even at the early stages of handling trimath some children will spontaneously have become aware that the shapes possess symmetry , some more than others. Their intuitive notions about this important property then undergo development through tasks such as these:

(A)



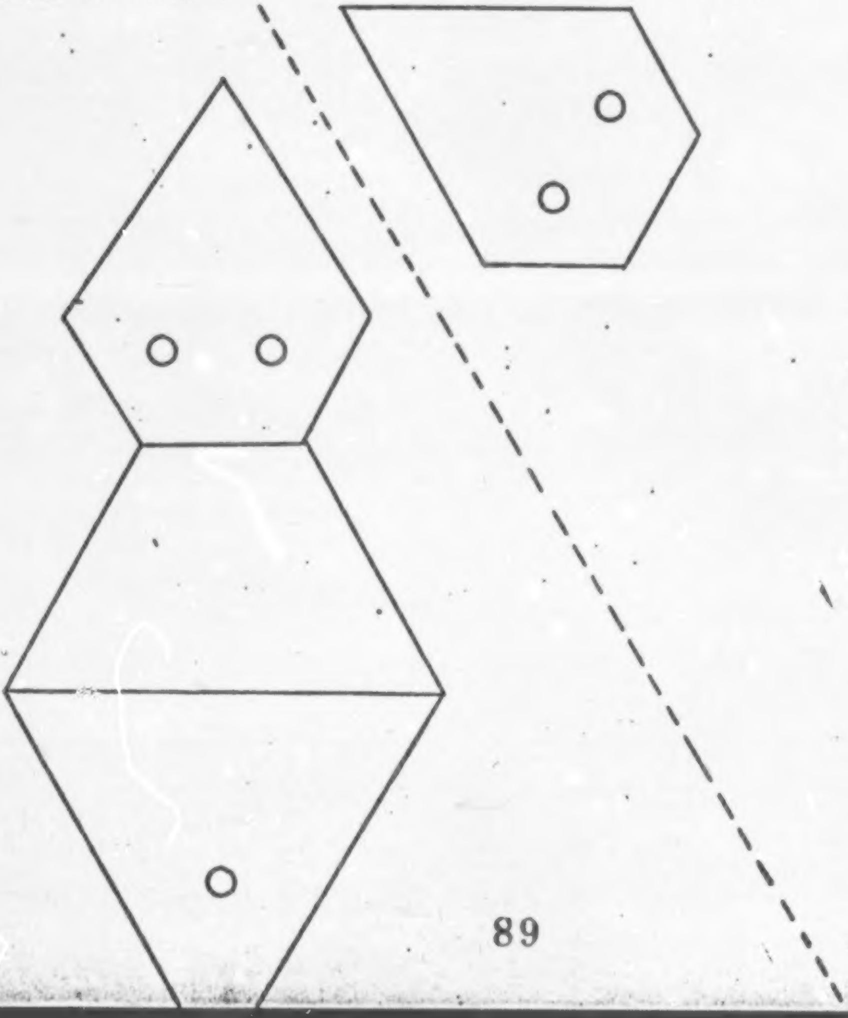
Take a mirror and place it along the dotted line of a triangle (any colour) with one hole. If you look in the mirror, you will see part of the figure reflected. Is this part the same as the part behind the mirror?

(B) Is the same true if you place the mirror along the dotted line shown here?

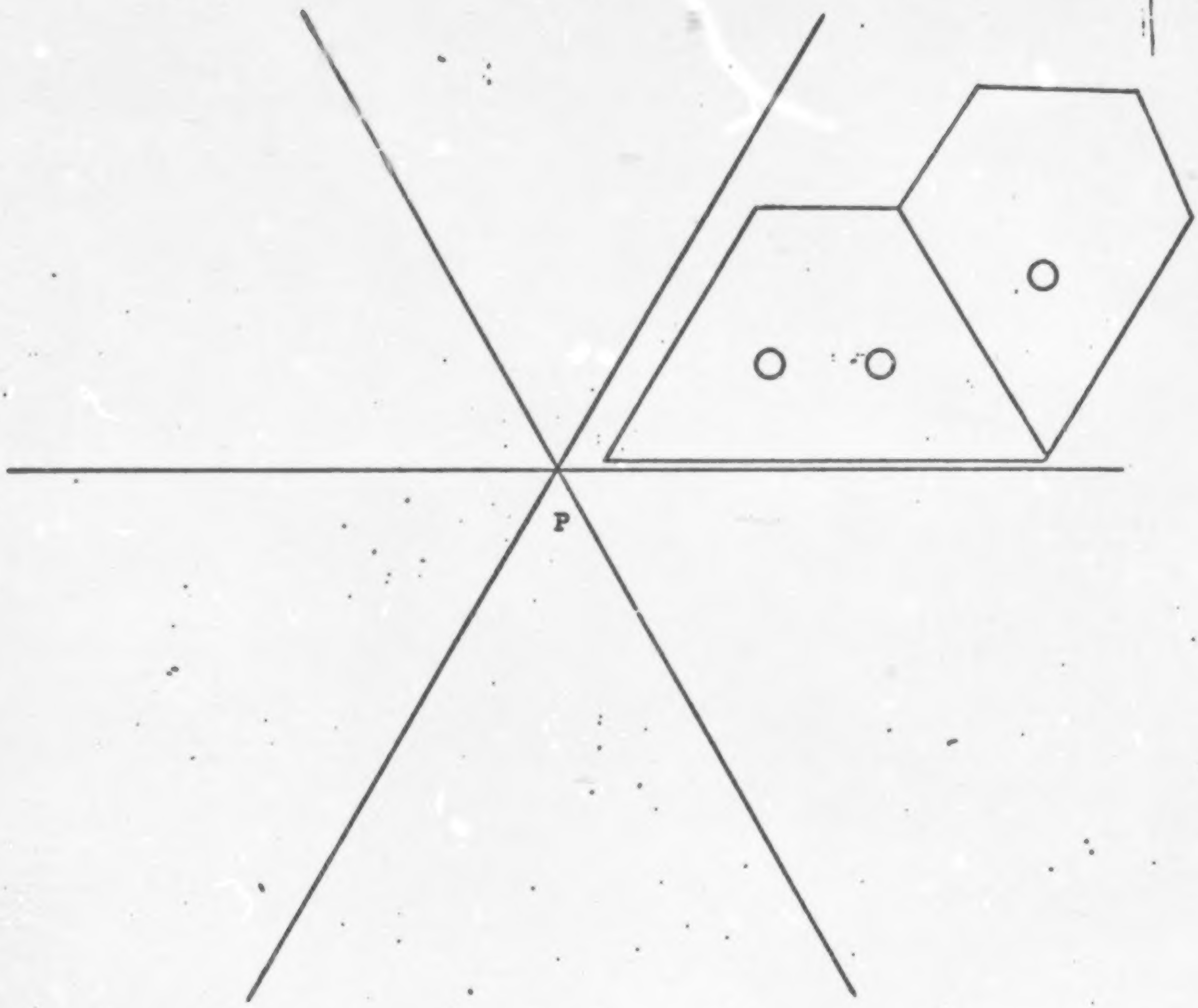


(C) Take some of the other shapes and find all their lines of symmetry.

(D) Complete the pattern on the right of the dotted line until it becomes symmetric about that line.



(E) Complete the following pattern until it has 60° rotational symmetry about its centre P.



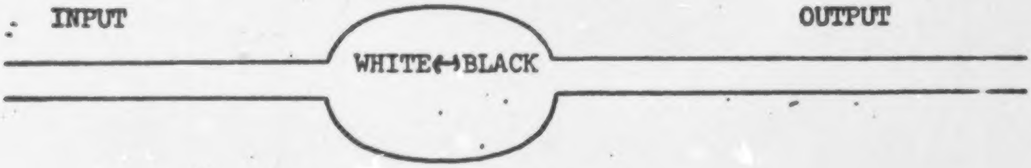
Change of colour destroys symmetry just as does a change in the number of holes. However if a pattern requiresmany pieces of the same kind the children will often agree to ignore colour. This trains them in accepting a somewhat unnatural convention and saves the inconvenience of breaking into several sets of the equipment on one table.

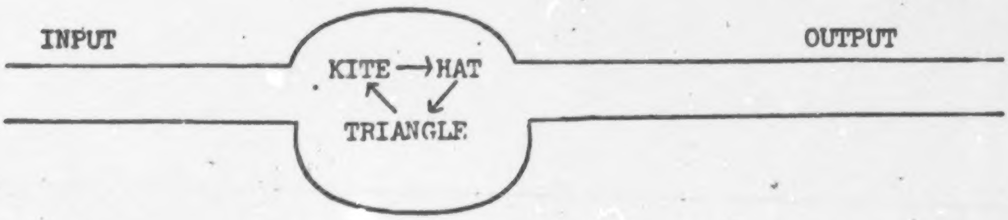
(v) Operators

An important feature distinguishing mathematics from other intellectual occupations is the use of operators. A machine which prints out enlargements of sketches represents such an operator, but the notion is quite general and applies in the abstract to multipliers, integrals or mappings.

For the children using trimath operators are commonly represented by drawings with verbal instructions on a large sheet of cardboard:

(A)





Operator (A) changes all white shapes into black ones and all black shapes into white ones if the output and input sides are interchanged. Grey pieces on the other hand it will reject, like a cigarette machine which has been built to take only certain types of coin.

Operator (F) will accept every type of piece. In one direction it will turn kites into hats, hats into triangles, and triangles into kites; if used the other way the kites become triangles, and so on. The order has changed from a clockwise to an anticlockwise one, but this was not noticeable in (A) where only two properties were exchanged.

Very elaborate networks can be constructed by chaining these and other operators together. This simulates the longer manipulations of traditional work and prepares the children to approach mathematical techniques intelligently.

Alternatively one finds these operators represented by plastic arrows of different colours. These can be moved about like the trimath pieces but contain no print. Their function is totally flexible and fixed by mutual agreement at the start of each experiment.

(vi) Order

Unlike our number system 1, 2,3. . . . ., in which 1 comes before 2 and 2 before 3 the design of the trimath pieces does not suggest any particular order in which they could be arranged. To explore this important concept the children are given or invent instructions such as:

(A) Triangles before hats, hats before kits,

(B) White before grey, grey before black,

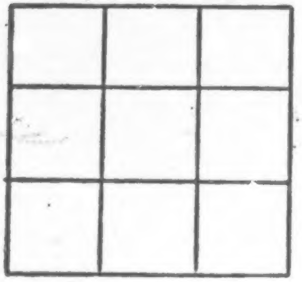
(C) 0 holes before 1 hole, 1 hole before 2 holes.

Even with these rules there is more than one way of setting the 27 pieces out in a row, since nothing has been said about the relative importance of (A), (B) and (C). To eliminate all ambiguity the children must add 2 further rules; for example, "(C) before (A) and (A) before (B)". To think of these is a more complicated affair since they are rules about rules, not rules about the pieces. If the children succeed they will have progressed to a second level of abstraction, a vital step in their preparation for detecting what is often concealed in traditional algebra.

(vii) Difference and similarity

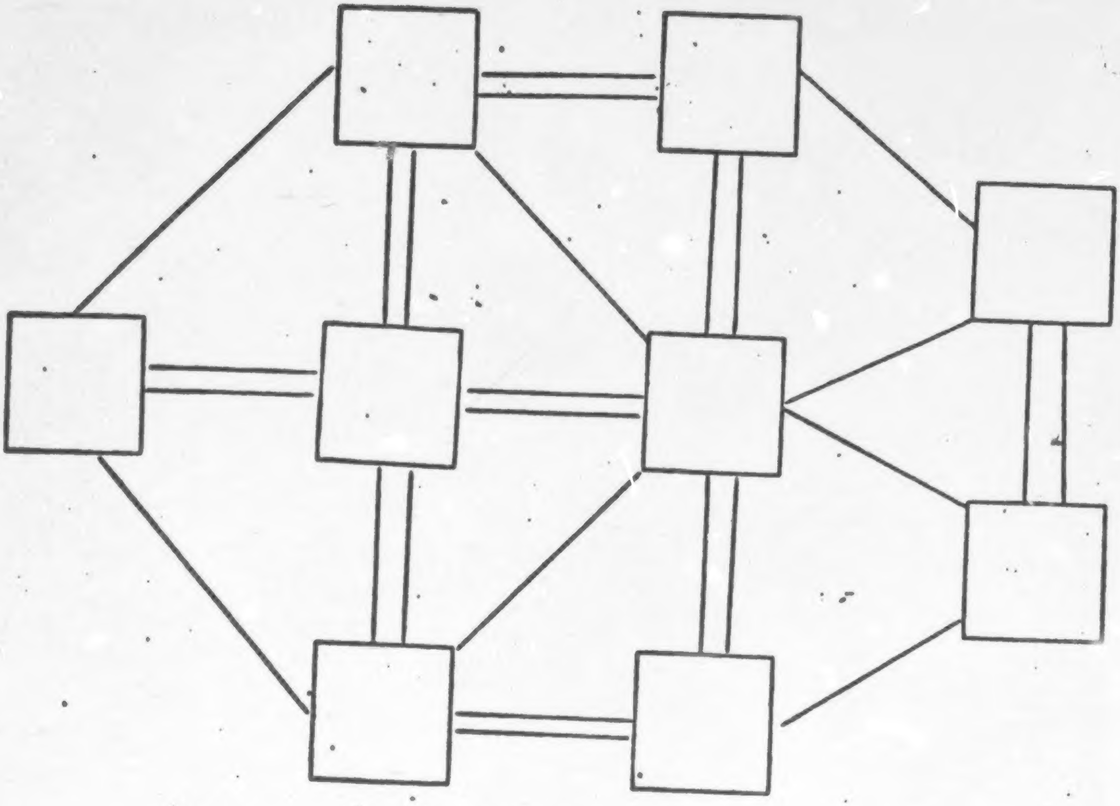
The process of classification is based on the fundamental relations of difference and similarity. Since any two trimath pieces differ from each in 1, 2 or 3 attributes a large range of exercises is open:

(A)



Use the triangles only and arrange them in the square so that two with the same colour or the same number of holes are next to each other.

(B).



Using the triangles, hats and kites without holes place them in the squares so that those joined by a single line differ in one attribute only and those joined by a double line differ in both colour and shape.

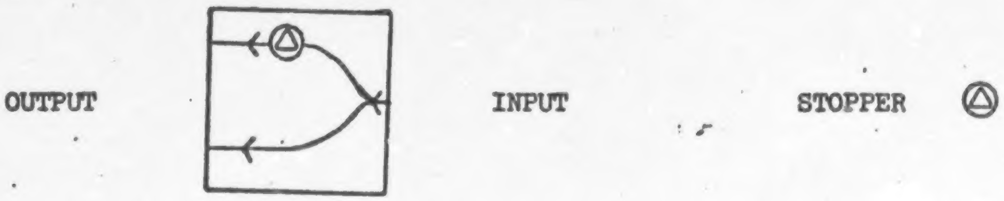
(C) The triangle has 3 vertices, the hat 4 and the kite 5. Each piece appears with 0, 1 or 2 holes. Add the number of holes to the number of vertices and you will obtain one of the numbers 3, 4, 5, 6, 7. Taking all the 27 pieces place those with the same number into a set (an equivalence class). How many different sets do you obtain and how many pieces are there in each set? (More sets can be obtained by assigning the numbers 0, 1, 2 to the three colours as well.)

(viii) The logic tracks

In mathematical situations one is often faced with an end result which has to be related to given information by several steps of strict logical reasoning. For children a concrete aid to such stepwise thinking is provided by the logic tracks. Each piece contains branchings of the track and there are also holes into which stoppers giving instructions on what is to go where are inserted.

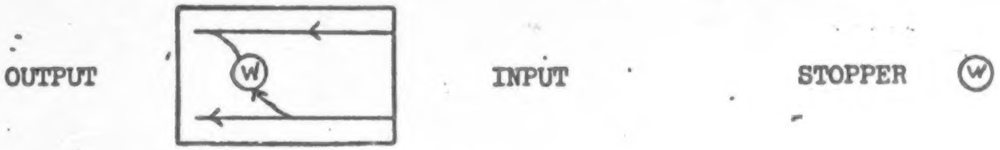
For example:

(A)



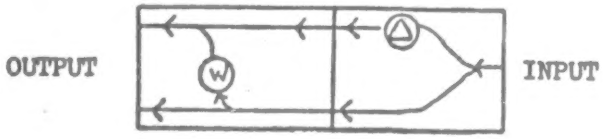
Encoded instruction: All and only triangular pieces must take the upper track, all other pieces the lower one.

(B)



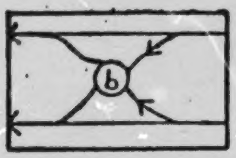
Encoded instruction: All the upper input goes to the upper output. From the lower input all white pieces are shunted to the upper output.

Suppose all the trimath pieces without holes are taken as input and tracks (A) and (B) are fitted together:



The upper output will now consist of all the pieces which are either triangular or white, or both triangular and white.

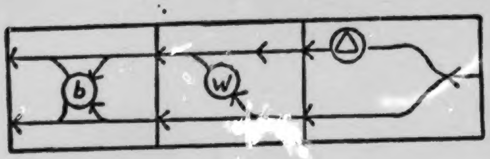
(c)



STOPPER ⓑ

Encoded instruction: All black pieces from the upper input are switched to the lower and all black pieces from the lower input to the upper.

If track (C) is fitted on to the combination of the other two we obtain:

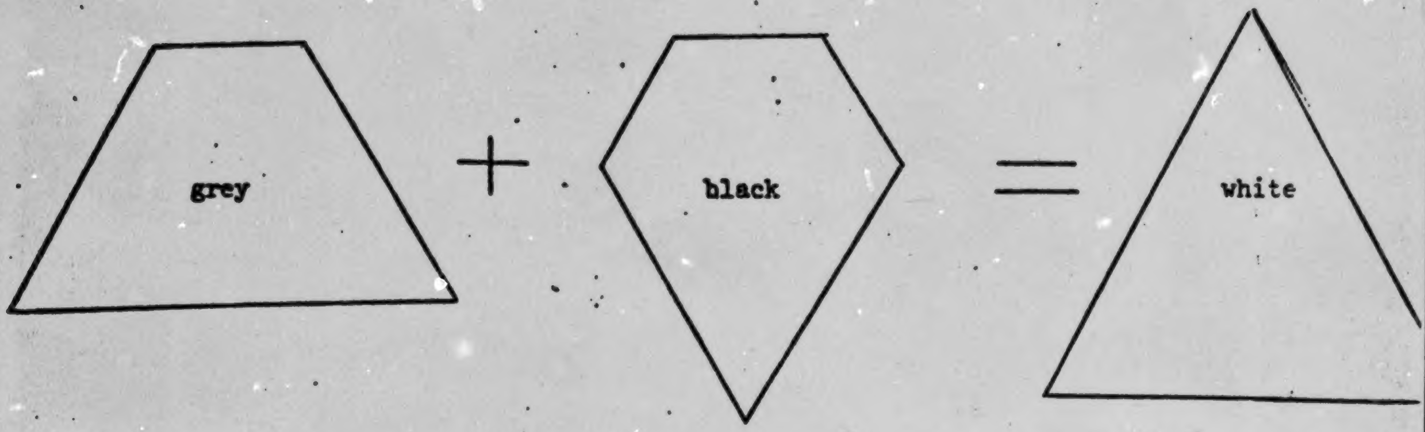


The upper output will now consist of all pieces which are grey triangles, or white; and it will also consist of all black pieces which are not triangles.

There are several other kinds of track. Taking various combinations of them makes it possible to put together a fairly complex set of instructions. Sometimes the children are given the final output and asked to find a combination of tracks leading to this output for themselves. If there are several possible combinations it is of serious interest to see which require the least number of tracks.

(ix) Little vector spaces

The reader will probably have felt that in the last three sections many of the descriptions taxed his thought processes more than did the earlier ones. This is because the visitor has been moving upwards and meeting with an increased concentration of mathematical content. Though still in the elementary school he has to work and ask his way around more and more. The hats, kites and triangles are still being arranged, but according to much more complex rules. Eventually he stands puzzled before a blackboard on which is written:



This equation only has meaning if one first understands the following convention:

COLOUR		SHAPE		NUMBER OF HOLES
white	0	triangle	0	0
grey	1	hat	1	1
black	2	kite	2	2

Since each trimath piece has colour, shape and a number of holes it can be completely described by a triplet formed from some combination of 0, 1 and 2. For example:

The black kite with no holes is described by (2, 2, 0).

The grey hat with no holes is described by (1, 1, 0).

The convention further stipulates an addition of these triplets. In this only numbers in the same position are added to each other, just as real and imaginary parts are added separately in the addition of complex numbers. Also there are only three possible numbers for each position so that the answer to an addition must never exceed 2. This is easily arranged by working in a modulo or clock arithmetic. 3 is equivalent to 0, 4 equivalent to 1 and 5 equivalent to 2, just as a railway time table treats 00.00 hours as equivalent to 24.00 hours.

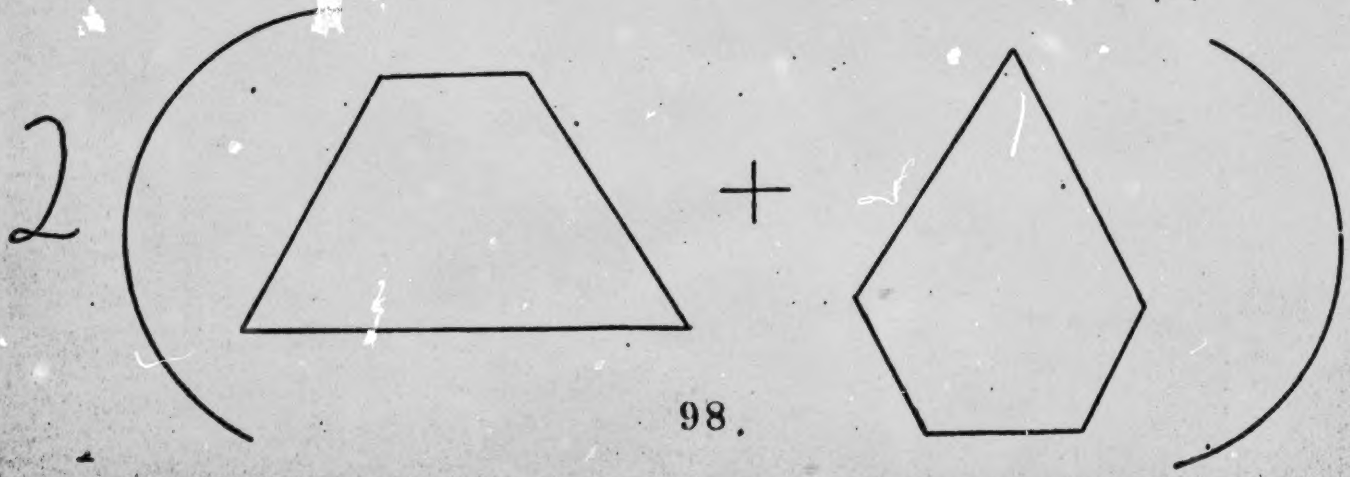
Written in triplets the equation now looks less mysterious:

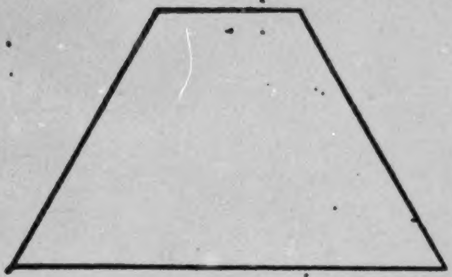
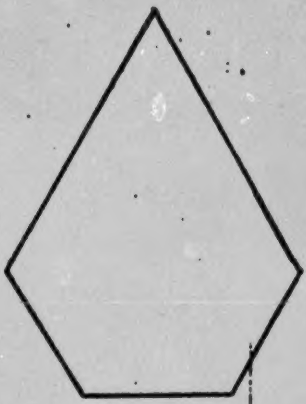
$$(2, 1, 0) + (1, 2, 0) = (0, 0, 0)$$

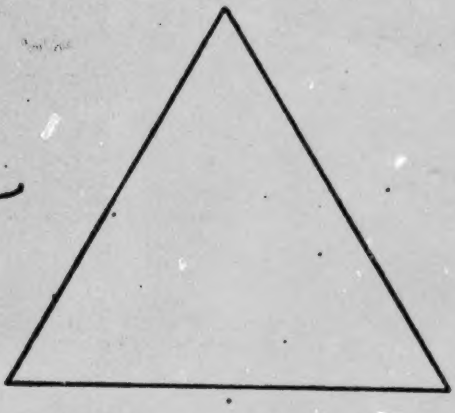
If it is also agreed that the triplets can be multiplied by any one of the numbers 0, 1 and 2 then the children are working in a system which is known to mathematicians as a vector space. An example of this multiplication would be:

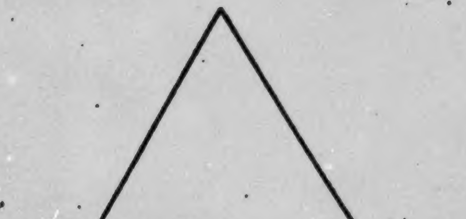
$2 \times (1, 2, 0) = (2 \times 1, 2 \times 2, 2 \times 0) = (2, 1, 0)$, since 4 is equivalent to 1.

With all this in mind the reader will be able to check that the following equation is both meaningful and correct:



= 2  + 2 

= 2 

= 

Much interesting work on subspaces, bases and transformations suggests itself. If from the 27 triplets the triplet (0, 0, 0) is excluded the remaining 26 may be interpreted as homogeneous co-ordinates for a projective geometry in the plane, with 13 points and 13 lines. From this again the removal of 1 line and hence 4 points will leave a 9 point affine geometry in which the missing line determines 4 directions.

3.9 Final Appraisal and Speculations

Here and now the new maths as developed by Dr. Dienes seems effective in the classroom. This simple fact speaks for itself, no matter how small the knowledge or how short the time the visitor has at his disposal. The children persevere without pressure, usually bring their tasks to a successful conclusion and often display unsuspected powers of originality in posing and answering problems of their own. There is no selection according to social status, no streaming according to ability. The approach is offered to the school population at large. Social and intellectual growth are fostered together in the democratic spirit which Dewey would have blessed. Of course one level of ability shows up against another in the speed, accuracy and creativity of the individuals who work and live together. But there is always something for everyone to learn, and to learn under conditions thought to be most conducive to his success. One hears of doctrines which simply divide men into those who can do mathematics and those who can not. If these need further corroboration they will not find it here.

To date materials and tasks have been developed as far as grade 6 and are undergoing [illegible] trials in grades 7, 8 and beyond. As far as one can judge the course is a comprehensive one. It brings children face to face with a very wide range of concepts as early as possible and ensures that their conceptual development advances towards the mathematics of the adult world on the broadest possible front. Against this gigantic network of organization one is tempted to view earlier efforts like those of Montessori and Stern as "special cases" in the slow revolution to present mathematics to children in a psychologically acceptable way. And the comprehensive nature of the undertaking also makes it difficult to conceive how the future could throw up a rival programme based on principles that were totally hostile or divorced.

What could be the long-term gains of working at mathematics in this way? Even if they retain no details the children will never accumulate that lasting sense of distaste for the subject to which even adults in intellectually demanding positions often confess. But the children's level of involvement is manifestly so intense that total obliteration seems highly unlikely. Instead we can expect them to retain a life-long curiosity and interest, no matter how small the mathematical demands their later occupations may make.

A more critical issue is the accumulation of knowledge and attitudes by those who will need mathematics as service subject in engineering, the sciences, or in the social field. If there is substance in recent criticisms that mathematics which is modern in either content or approach enfeebles proficiency in the traditional techniques the Dienesian programme takes care to accord these techniques their rightful place. More than that, they be motivated by genuine understanding of those principles from which they derive. There is evidence to suggest that full understanding reduces the time needed for practice; add to this the manifestly more positive attitude to learning and the way into this or that piece of service mathematics should be straighter than before.

Finally there are those who will seek to advance to the frontiers of the subject in some area of mathematics, pure or applied. A small percentage among them is so highly endowed that they will advance to the top inspite of teachers, textbooks or courses. Immediately below them is a substantial number for whom the teaching strategies at school or college could make all the difference between failure and success. At the present time the Dienesian programme is just extending its feelers into the middle school. Conclusive evidence of its faults and merits may be lacking for many years to come. This need not stop us from raising two general questions to which mathematics education in the large might one day like to have an answer:

(1) Will the approach adopted here endow students at school with sufficient mathematical insight and expertise to render them immune to bad or indifferent pedagogy later on?

(2) Could the approach be extended with profit to all levels of mathematics learning?. In theory the principle of multiple embodiment as instanced in the wooden blocks or plastic shapes should facilitate understanding, no matter where it is applied. What must of course change is the type of embodiment. In higher learning concepts are generally abstracted not from blocks, colours or shapes within our everyday experience but from mathematical objects that are already highly abstracted themselves. Imagine a mathematician experienced in numerical analysis who wished to make some acquaintance with the highly abstract branch of modern mathematics known as category theory. What should

the introductory pages of a truly instructive text on this subject contain? To the categorist the ignorant reader stands in very much the same relation as the grade I child to his teacher. If there is a general solution lurking in the Dienesian approach then it might ultimately make communication among mathematicians less problematic than it seems to day.

Chapter 4

The New York Project

The OPI Project in Hungary and the Sherbrooke Project in Canada each operate within several schools, all of which belong to the public sector of education. There is no question of any social selection of the students, or the payment of fees. Generally students attend because the schools are conveniently situated near their home, not because their parents wish to send them there and are financially privileged to be able to make this choice.

Of the New York Project the exact opposite is true. It operates in a single, independent school which was founded for a section of society in which parents have both the desire and means to support an unusual venture. They sought to fuse the best in American and European educational tradition and out of this quest the Fleming School was born.

4.1 Historical Sketch of the Fleming School

The Fleming School, through which a Dienes-type programme has come to be established in the USA, was founded in 1957 with 30 students and 4 teachers in the confines of two rented rooms. It was to provide early education for children aged 3, 4 and 5, that is over the three years before the public sector could admit them to the first grade. The success of this early immersion was so startling that parents were reluctant to move their children when they reached the age of transfer at 6. Pressure to widen the school's range increased and a large programme of expansion was set in motion. Today the enrolment of the Fleming School is around 500 and all classes as far as grade 8 are housed in two large buildings, where space and equipment (Fig. 4-1) permit both teachers and students to enjoy the kind of educational setting that was planned. Those who leave after completing grade 8 have been known to obtain places in the best High Schools of Manhattan or in Preparatory Schools scattered over the country at large.

(Insert Figure 4-1 Here)

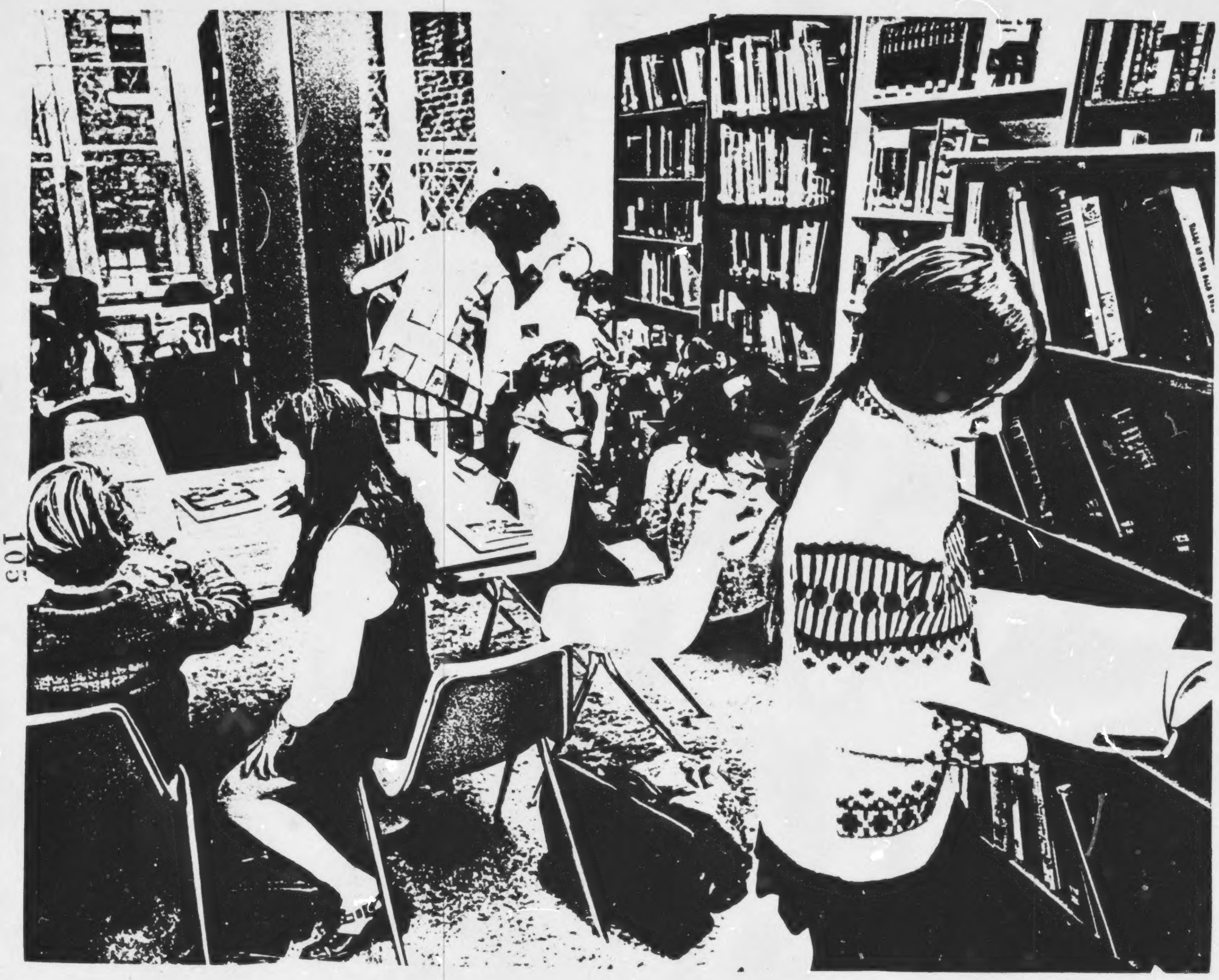


Fig. 4 - 1 The library of the Fleming School

The Fleming School is chartered under the Board of Regents of the State of New York, but financially and otherwise independent of any School Board. It is a member of the New York Independent School Guild and maintains all necessary relations with external examining bodies and the public sector of education in general.

4.2 The Philosophy of the Fleming School

The outlook of the Fleming School is progressive, both as regards student-content and student-teacher relationships. It is held that the child is capable of benefiting from education in an institution at an early age, provided that the environment in which he is placed is structured in a way that will promote the growth of his personality as a whole.

The student-teacher relationships must be of the kind that will promote confidence and self-assurance, but at the same time also a spirit of mutual co-operation rather than selfish competition. Without being dogmatic or authoritarian in a conventional way teachers must seek to make the youngsters aware of the social constraints surrounding their existence. If this is done in the right manner the children will come to accept these restrictions as something natural and realise that the sacrifice of individualism brings rewards in its train.

The student-content relationships must be adapted to the stage of cognitive development which the young person has reached. Every child is curious, curious to learn more and more about his environment (Fig. 4-2). In a progressive school this curiosity is fostered and enlisted as an essential constituent in the learning situation; it provides the dynamics of this situation by creating the motivation essential for the assimilation of content.

insert fig. 4-2 here

Stress is laid on discovery learning. The teacher provides a large collection of tasks whose complexity he regards suited to his students' age level. This provision takes the form of concrete materials and written instructions. As the child thinks about the instructions and materials he becomes aware of relations other than those to which his

Fig. 4 - 2 Playing with the Dienes Multibase Blocks

attention was directly drawn at the outset.' This growth of awareness beyond that which is directly given is what constitutes discovery. The child finds new relations in the material if his mind is ready to find them. For this reason his discoveries are "natural" to him and will be more easily remembered than if they were just the discoveries of others, neatly packaged in some textbook language and presented to him in some unpalatable form of drill.

Stress is laid on creativity in both individualised work and group projects. There is creativity in art, music and dance programmes, creativity in the production of films and plays, and also creativity in the strictly intellectual pursuits of the science laboratory and the mathematics class. All creativit[illegible]ems from a desire to produce. The desire to attain a chosen objective acts as a continuous motivator to search for the means of achievement, to examine what one has found, to discard the irrelevant and to use what is left as a means for moving nearer to one's goal. In the search for the means the teacher and fellow-students count among the resources that can be consulted (Fig. 4-3). This fuses the social with the intellectual aspects of the situation and goes a long way towards creating that overall harmony and happiness known as the "spirit" of the Fleming School.

Insert fig. 4-3 here

4.3 Bilingualism

The majority of non-American students at the Fleming School come from French families and the American students generally come from families whose experience of world travel has convinced them that knowledge of a foreign language is a valuable asset. These are among the considerations which have led to the introduction of a bilingual English-French programme commencing at age 3. Instruction is fairly evenly divided between the two languages and by the time a child reaches grade 3 his use of the French language comes quite natural to him.

Utilitarian considerations apart the study of a second language enriches the field of possible cognitive experience that is made available to the child. If sentences written

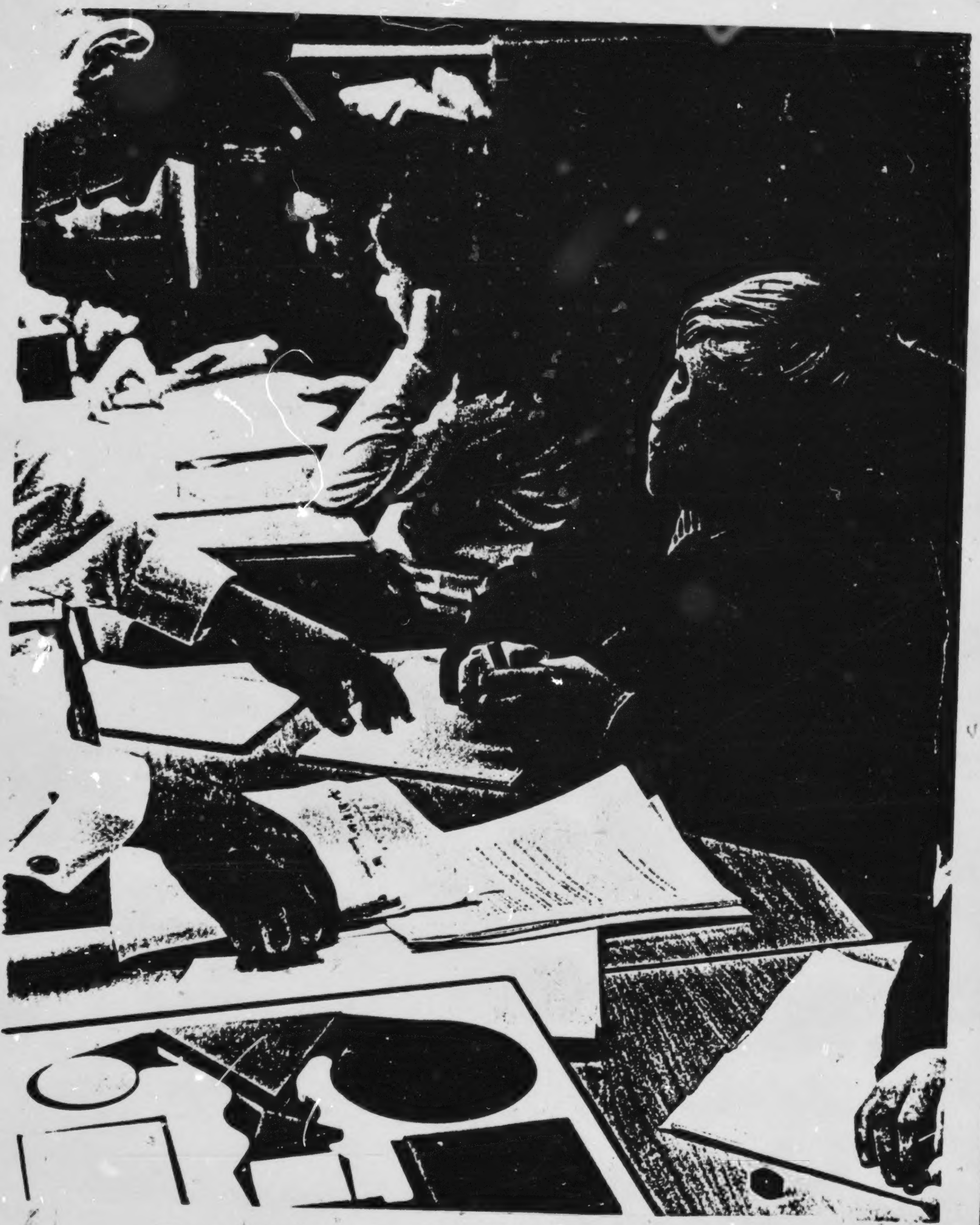


Fig. 4 - 3 Discussing the peculiar aspects of a discovery

in two different languages express the same thoughts then they are, in Dr. Dienes' terminology, two embodiments of these thoughts, and the learner's changeover from one embodiment to the other facilitates his understanding of the thoughts for their true meaning. The practice of bilingual instruction, which is becoming more and more popular in the English-speaking schools of schools of the province of Quebec in Canada seems to be adding a new and important dimension to the curriculum, one that could not easily be provided by other means. Learning the second language as a side-show in parrot fashion is one thing, living out half one's school life with the second language as the chief means of communication quite another.

4.4 Teacher Selection and Preparation

In keeping with the cosmopolitan spirit underlying its foundation the Fleming School has also adopted a policy of recruiting its staff in those parts of the world where the best teachers of any specialism are to be found. The French portions of the curriculum are taught by teachers from France, and the subjects of English and Physical Education by teachers from England.

It seems generally agreed that the School's unconventional policy has produced satisfying results. Its approaches are rewarding but at the same time very demanding, especially on the teacher who has to break away from earlier habits and re-think his way through new aspects and new presentations of his subject from scratch. As a rule the children work in groups and each group works at its own rate. The teacher has to think of the needs of each group - what they are doing to-day, when they will finish, and what they will need to-morrow - and as far as possible also of the needs of each individual within the group. The faster learners need supplementary tasks of a more challenging nature, the slowe ones repetition and many different presentations of what are essentially the same tasks. Only when a certain level of self-reliance and mutual trust has been established can he expect to go about his work unimpeded, and to establish this level he must in the first place have laid his foundation stones correctly.

One often hears it said to-day that with major changes in the attitudes towards authority the image of the teacher in the classroom also is no longer what it was before. In the co-operative atmosphere of a progressive school he can scarcely remain omnipotent and certainly not omniscient. The children no longer merely wait to be asked to repeat what he told them on the day or during the week before. Instead they come up with their own questions and expect more than an evasive answer. If the teacher does not know he must be prepared to admit this there and then, adding a promise that he will assist with the search for an answer in due course.

4.5 The Assessment of Progress

Sooner or later any programme which breaks with tradition faces the problem of meeting up with other streams of educational development at the point where it ends. During their years of attendance at the Fleming School students do not take public examination Instead they are at regular intervals given tests of reading, achievement and intelligence. When these tests are corrected by the Educational Records Bureau their results are communicated to the School. Taken all in all these results, which reflect an extremely narrow sample of just cognitive variables, have never given any cause for concern. It must be stressed that they give no indication of the school's spirit and all its possible side effects, so that in any comprehensive evaluation they could only form a small component alongside the actual performance of leavers in their later education and beyond.

4.6 Initial Contact with the Dienes Method

In 1967 a teacher from the Fleming School read about the Dienes method and decided that its principles were in complete accord with the philosophy of his school. To make closer enquiries he decided to pay a visit to Dr. Dienes' headquarters at the Research Centre in Psychomathematics at the University of Sherbrooke. On his return to New York he convinced the school's director and several other teachers that a mathematics programme as developed in Sherbrooke would be ideally suited to their requirements. Even in mathematics it was possible to adopt a child-centred approach without jeopardising one's obligations to the intellectually demanding structures of the subject. In fact the new method advocated that the children face the challenges of logical thought from the earliest stage, and the means for getting them to do this could be tailored to the level of mental development they had reached.

4.7 Adoption of the Dienes' Method

In the following year (1968) the Fleming School and the Sherbrooke Centre signed a formal contract. In this contract provision was made for key teachers at the School to study existing programmes at some of the Sherbrooke schools and also to undergo some systematic training at the Centre themselves. Dr. Dienes in turn agreed to hold regular workshops for the teachers and to visit the Fleming School in person several times a year.

It did not take very long before the plans for co-operation between the two sides began to be translated into action. That the reaction of members of the staff at the school was one of unaffected frinedship and gratitude is shown in the following letter from the School's Director to Dr. Dienes:

"I must tell you how happy and enthusiastic our teachers were about the workshops at our school last Friday.

Miss W. and Mrs. L. are infinitely grateful to you and want to thank you for all your solicitude during their stay in Sherbrooke. They both came back radiant with enthusiasm about what they had seen and they are really filled with admiration for your work.

May I take this opportunity to remind you that you have so kindly promised to make a three-day visit to our School on November 18th,19th and 20th. We are all looking forward to this with great expectation and pleasure.

I believe Mr. F. had promised to get some record cards from your secretary to send to us by Mrs. L. or Miss W., but unfortunately forgot. I wonder if you could possibly forward some to us as I want to further the present enthusiasm of the teachers to the fullest extent."

It was an example of the frank rapport so characteristic of members co-operating in the various Dienes projects. Aware of Dr. Dienes' tight schedule of commitments here, there and everywhere the Director put in a reminder about the next workshop. Dr. Dienes replied re-assuringly and sent ahead sample questions on which his workshop would be based:

Rules for sequences.

Start with very simple sequences such as

& % % & % % & % % & % % & ....

and ask the children to go on.

Proceed to more complex rules such as

A B A B B A B A B B A B A B B A..........

or P Q R p p q q r r P P P Q Q Q R R R p p p p q.............

In some cases it is enough to know one element in the sequence in order to tell what the next element is going to be. In other cases it is necessary to know two or three elements.

With little children do not attempt to get them to verbalize the rules. If they can continue the sequence faultlessly without any explanation, it is enough.

With older children, from about 9 or 10 onwards, it is possible that they can formulate the rules precisely and apply them.

For example, the sequence

1, 2, 5, 6, 9, 10, 13....................

can be "verbalized" by saying

"If it is odd, add one, if it is even add three, to get the next element"

Children can play games with each other by giving each other more and more complex sequences, for which they have to guess each other's rules.

For example, a harder sequence would be

4,5,6,2,8,3,10,4,3,5,14,6 ....

A rule for this sequence would be

"If the number is prime, add three; if it is not prime, divide by the smallest factor" .

We can also use sentences or words out of which to form sequences.

For example:

"The cat ate the mouse, the mouse was eaten by the cat, the cat ate the rat, the rat was eaten by the cat, the dog ate the mouse, the mouse was eaten by the dog,.............. "

What are the two missing sentences ?

What is the rule for writing the "next" sentence ?

In this short extract taken from what was actually sent there are challenge and surprise as one passes from one line to the next. Here and there some basic knowledge of numbers is presupposed but several of the questions can be answered from the thoughful analysis of knowledge that is much more general. An important objective of the method is reflected, namely the widening of the range of activities which teachers have come to associate with mathematics at the elementary level.

Some time later a parent only superficially acquainted with the theoretical backgroun to the new mathematics method but not slow to see what it was doing for his oldest child came out with this appraisal in a letter to Dr. Dienes:

"My own work has moved away from mathematics education; I'm with a firm doing different kinds of anti-poverty work - in welfare, health, employment and the like.

My oldest child, however, is now in the kindergarten at Fleming, playing with attribute blocks as if they were always part of her, I presume ! I haven't had much direct contact with Fleming, but through a series of circumstances I was asked to write an article about the math program to help Fleming parents better understand what is of parents, and there hasn't as yet been an opportunity to work out a real parents' education program.

The editors of the parents' paper are very interested in having something printed which will assist the parents. Hopefully, this article will do this and at the same time do justice to your program. Most of the ideas come from Building Up Mathematics, as you will plainly see.

I do hope it is satisfactory to you or that it can be made so with only minor changes. From the little I've heard I'm sure that more information for the parents is a good idea.

I look forward to seeing you when you come later on in November. The newspaper will come out after you are here, probably at the end of November, so it could be a good follow-up to your visit. I hope that Sherbrooke is suiting you and that your family is well."

It was the typical reaction to something that could be seen to be doing much good and yet remained strange and puzzling - like the first message sent by cable or the first carriage that moved under mechanical propulsion. In an independent school the parents are as a rule more intimately concerned with their children's progress. Dr. Dienes has never failed to realize that they are everywhere involved as a third party, even if they are not as regularly and readily available as the children and their teachers. He agreed to hold a parent's meeting at which questions of doubt and requests for elucidation could be put to him from the floor. Whereas it would be vain to expect that the transactions at these meetings reach all the parents who do not attend and leave all those who do attend convinced, such open discussions with the world beyond the classroom walls are an integral part of the service which the Sherbrooke Centre provides. It has been pointed out that the voices pronouncing on mathematics education today are many. There is nothing to prevent parents from being exposed to some of these more than to others. Not working in the Dienes Project themselves they then only have the occasional meeting and the general impressions of their children's reactions to assist them in forming a judgment.

Through this policy of give-and-take the Dienes and Fleming School programmes continued to identify their interests from year to year until in 1972 the new mathematics method was regarded as firmly enough established to be subjected to some formal evaluation.

To understand why educational changes are slow - much slower, than say, the conversion of a fleet of aeroplanes from turboprop to jet - one has to consider the amount of adaptation that is involved on the human side. The teachers have to discard old habits for new ones. Most of them are not trained in mathematics to a high level and must in this condition come to terms with the demands of a programme that is as intellectually demanding as it is profuse. Some will visit the Centre in Sherbrooke and then tell the rest what they saw and learned. All have to sit down and try their hand at a wide range of unfamiliar activities before they will feel confident to try them with the children. As the children are always present the teachers' familiarisation has for the most part to take place concurrently with the initial try-outs by the children. It may take months and even years before a teacher can claim to communicate the ideas efficiently, and this time will be lengthened if his teaching load consists of other subjects besides. The Fleming School differs from elementary schools in the public sector in that its teachers are specialists, that is, they take their subject from one class to the next instead of staying with the same class throughout the day. This has some drawbacks but also favoured the rate at which knowledge of the Dienes' programme could be assimilated. But on the side of deficit there is the flux in staff. Newcomers who are brought in to fill the gaps of those who have left must themselves go through the whole process of adaptation from the start.

On the children's side there are also difficulties. All but the youngest of them have started with mathematics in a certain way and already adopted some orientations of like or dislike towards the subject, also certain ways of thinking about its rules and remembering them. The new approach aims to provide them with greater stimulation and incentives to think about mathematics more efficiently, but this can not overnight give a child of 8 all the experiences he would have had if he had started with the new method at age 3 or 4. In the Fleming School as in most other schools the Dienes' approach tried to reach every grade as soon as possible after it was approved by the authorities concerned, but this does not alter the fact that the majority of those attending in 1968 never reaped its benefits to the full. Children, it is true, change quickly and have marvellous powers of compensation when it comes to making up for lost phases in development. But the wealth of potential experience in the Dienes programme is so large - even in the first few grades - that anything more than partial compensation must be regarded as an impossibility. Added to this we have the fact that in 1968 the teachers' level of competence was far from established and that there would be no one capable of distilling and condensing the essence of the programme's earlier stages for the children in the middle school.

It is significant that the evaluation projected late in 1972 and actually performed in 1973 concerned itself with students in grades 4 and 5, that is with students who were in grades K and 1 at the time of the Dienes Project's initiation. It was thought that these children had enjoyed exposure from a reasonably early stage and over a period of sufficient length to warrant the assumption that the effect of the new method on their mathematical outlook and competence would be substantial. Some 6 months before this evaluation Dr. Dienes summarised his impressions to the Director:

> " As a result of my last visit to the Fleming School let me make the following observations:
>
> 1. Although the standard of motivation and of general intellectual involvement is high in almost every case, the classroom organization still falls short of the desirable, mainly as far as the physical location of the working groups is concerned. This is due -
> (i) to the fact that you have specialist teaching and so the classroom has to be re-arranged before and after each lesson, and
> (ii) to the fact that the size of the classrooms is simply not adapted to a very active situation, i.e. the rooms are much too small.
>
> Now that you are acquiring new space, would it not be possible to arrange things differently?

I suggest the establishment of as many maths classrooms (laboratories). as are necessary to house all those classes which have too little space. This would solve both the problems (i) and (ii). You would probably need a lab for every five or six classes.

Furniture could then be provided for storing the materials, instruction cards etc. in an easy filing system, tables could be used instead of desks and more blackboard and other feltboard space could be provided all round the room.

2. Although some of your teachers are now fairly knowledgeable in the maths involved, there is a great need for a full time maths co-ordinator. A person who has the mathematical knowledge as well as the pedagogical expertise to run such a program is extremely rare as yet. I had hoped to train such people at Sherbrooke. If we are lucky enough to obtain the services of such a person we must make his coming to work in New York worth while.

3. The evaluation will be done in February. This will include not only mathematical knowledge, but "learning ability" tests, a problem facing test, anxiety measures etc. so as to obtain as complete a profile of the school as possible, in comparison with a control school. Dr. K. is an extremely able psychologist who has already done such an evaluation for the Hungarian Project. He is working in Sherbrooke now and is about the only man in existence who has
   (i) the practical and theoretical psychological knowledge required
   (ii) an intimate knowledge of the theory and practice and the underlying philosophy of a "Sherbrooke type" project to be able to ask the relevant questions and obtain sensible answers. You will be getting all the information in January about how to use and enrich our item bank for testing. I can discuss this with you on the 20th December."

4.8 An Attempt at Evaluation

The evaluation just referred to was carried out during a 10 day period in April 1973, a little later than projected. It formed another member in a series of similar evaluations that had taken place in Hungary and in Sherbrooke.

Five questions were posed and answered on the results of tests given to students of all ability levels in grades 4 and 5.

1. In traditional mathematics students of the Fleming School were superior to those of a Dienes-type school in Sherbrooke.
2. In modern mathematics the same students were only marginally better.
3. Students at the Fleming School have a very positive attitude towards their teachers and their lessons.
4. The standard of learning ability is high compared with the school in Sherbrooke.
5. Taken all in all the attitude towards problems is less favorable at the Fleming School than might have been expected.

These results are no doubt in part attributable to the general atmosphere existing at the Fleming School. In no way do they seem to bear out the contentions of critics who maintain that a modern programme of the Dienes type enfeebles performance in mathematical skills of the traditional type. Had it been possible to administer these same tests to a traditionally orientated school of similar socio-economic standing in New York then the case might have been even more convincingly refuted. Arrangements for such a comparison were made but suddenly cancelled for reasons outside the experimenters' control.

The reader will appreciate that to obtain decisive answers in such matters is a lengthly, involved and costly business. On the informal side one could ask him to consider the satisfaction which both teachers and taught claim to have derived from the new method, also the high standards reached in external tests in algebra by those who leave after the completion of grade 8; they are often given a year's advance credit in this subject when they reach their high school class.

This evidence may do no more than convince the reader that the case is "not proven". If it does then let him rest assured that means for obtaining a more favourable verdict are still being sought.

4.9 The Situation To-day

Six years after its first exchange the contract between the Fleming School and the Sherbrooke Centre remains in force. Much of what was then a blueprint has become reality. Two specialist co-ordinators have been appointed and all classes now use or are strongly influenced by the Dienes approach. The three upper grades use a supplementary textbook in algebra from another author, but this does not mean that they have discarded the Dienes approach for what it offers in the way of thinking and living together. Although no compulsion is applied the efforts of two special co-ordinators for the method have made it possible for all teachers to participate and none have shown themselves reluctant to try the new materials and exercises as they reach the School. The transplant from Sherbrooke has here found a natural home, one in which even cursory inspection shows that all are happy to have been involved.

4.10 Some Misgivings

Parents continue to express misgivings about what is happening in the name of mathematics. Without going to the effort of a closer investigation they see no link between the games with coloured arrows, triangles and squares, and the calculations they have unconsciously come to identify with the whole of mathematics. Their misgivings have been endorsed by some poor performances in conventional computation tests in the middle section of the School, and by the hostility expressed towards the "New Math" in some recent literature (Kline, 1973).

The narrow identification of mathematics with numerical calculation is one which the Dienes approach does not and can not make. Number work has its important and rightful place at many points in the Dienes curriculum. But it is introduced as the expression of intelligent thinking rather than as conventional practice with large calculations. To meet the requirements of a traditional computation test at a stage set by fashion rather than any profound psychological considerations would entail a break with the carefully planned hierarchical structure that characterises the programme as a whole. Speed at routine calculations may be an objective of primary importance for a student about to take on employment in statistics or banking. This happens long after the date for which the tests here under consideration were fixed. There is some evidence that children who are trained over-early for this kind of performance tend to develop a narrow attitude towards numbers and sometimes come to regard them as somehow lying outside the scrutiny of logical thought. Unfortunately the Dienes method has not been long enough in operation to allow one to say categorically that students brought up throughout their school career in its stimulating atmosphere can learn such calculations easily and quickly when they know they are really required. Were this to become established as fact then of course all fears except those of the fashionable but ill-timed examination would be dispelled. At the moment the overapprehensive parent, it seems, must be asked to look at the method as a whole, at the positive contribution it makes to the attitudes, discussion and performance in the classroom. Many of the questions spontaneously posed by children would be totally meaningless to comparable children brought up in mathematics along traditional lines.

As regards the hostile literature this has generalised its venom against new mathematics as a whole because some new mathematics has been found unsatisfactory. Over the past 15 years many programmes constructed on widely diverse principles and overall objectives have sprung up throughout the world.* All have been called new, but closer examination often shows that the attachment of this adjective is nothing more than equivocation as far as their inner structure is concerned. The Dienes programme is new, but there seems little justification for judging it on samples of students that have been taught some new method elsewhere. In any case, these attacks for the most part offer criticism without construction. To find dissatisfaction with the approaches of 30 or even 20 years ago one has only to look at the articles liberally scattered over the journals of mathematical associations in North America and in Europe. In the absence of any substantial body of positive suggestions the aims of today's neo-traditionalists must remain a matter for conjecture. Even if they reject the methodology as put forward in this book they have still to contend with the new branches of mathematics that are influencing the lives and activities of people who are not mathematical specialists. As an obvious example the human calculator is being replaced by the mechanical or electronic computer on an ever-increasing scale. This would seem to create a demand not so much for the speedy and accurate execution of routine calculations in one's head as for the ability to understand what these calculations are about, how the outputs of these machines may be estimated and checked.

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* See Appendix 1.

CHAPTER 5

EVALUATION

5.1 The Need for Evaluation

A "change agent" who introduces a "planned change" - that is, a change which is derived from a purposeful decision to effect improvements in a personality system or a social system and which is achieved with the aid of professional guidance - must receive feedback concerning results of such a change. There is no reason why educators should be an exception to this rule (J.D. Williams, 1971). The growing number of new programmes in practically all subject matters demand in the way of feedback more studies for the measurement of the effects of different teaching materials, instruction, and curriculum. This need is especially evident in mathematics learning (see e.g. Merwin, 1969; Reys and Post, 1973). Because of the radical changes in the curriculum at all grade levels, coupled with a variety of new pedagogical techniques, the evaluation of mathematics programmes is considerably more important today than was the case a decade ago (Begle and Wilson, 1970).

Without an evaluation process, the innovator, even in the most capable case, can express only his own feelings; for example, as W.P. Hull (1963) does: "We can only say that, intuitively, we feel that we are on the right track," or as R.B. Davis writes: "We do not know whether we are on the right track or not ....". Obviously, personal conviction is in certain cases not enough. In most countries, before any new method of education can be used to any great extent, there must be some "objective" proof of its advantages. Stake (1967) emphasizes this in his explanation of M. Scriven's (1967b) opinion that the evaluator is the one who should judge education systems: "Different decisions must be made - P.S.S.C. or Harvard Physics? - and they should not be made on trivial criteria; e.g., mere precedent, mentioned in popular press, salesman personality, administrative convenience, or pedagogical myth." Therefore, it is imperative that we face the problem of evaluation in order to successfully confront the problem of improving education.

5.2 Difficulties in educational evaluations

While the necessity for comprehensive evaluations is greatly increasing, it is also true that those educators who wish to carry out evaluations are very often confronted with several obstacles (see J.D.Williams, 1963; 1965; 1967; 1971). For example, teachers' abilities (their experience with the method used, their enthusiasm or otherwise for the method, etc.) are liable to influence results considerably; and the effect of experimental stimulation is difficult to control, especially in short term investigations; matching pupils in experimental and control groups in terms of mathematics performance is a particular problem; and if the change introduced is a complex one (such as the simultaneous change of both method and curriculum), then it is difficult to isolate the factor which causes the result. Unplanned changes can take place in the experimental situation, and these must be taken into account. The instruments of evaluation can turn out to be either inappropriate or without known application. Throughout the evaluation process it is possible to overlook some special merits of the educational project, or, on the other hand, to discover trivialities about it. The more noteworthy student achievements are unlikely events and are easily masked by statistical noise in testing procedures. In education - as in quantum physics and brain surgery - the act of measuring causes a significant alteration of the situation.

Evaluation is primarily concerned with changes in individuals or in groups of individuals (B.S. Bloom, 1970). The measurement of change presents numerous unsolved problems (see e.g., C.W. Harris, 1963), especially in scaling and reliability of testing. For example, the reliability of gain scores are mostly sufficient for research conducted on groups but rarely for the purpose of individual measurement (J.P. Guilford, 1970). In most school operations, the experimental materials are often used only by the better teachers with above average students and are not adopted on a wide scale until an explicit evaluation design is accepted and followed by the school (Begle and Wilson, 1970).

The inability to solve these and other difficult problems is partly responsible for the present tendency to spend even less effort on evaluation than some years ago. While some psychologists have lost interest because they cannot control the variables closely enough many "educationists" have become dissatisfied with evaluation processes because of their "sterility". There is certainly a tendency in psychological experimentation toward increasingly trivial meaningless investigations which are irrelevant to human problems (see R.N. Sanford, 1965). As Varga (1968) summarizes: "Paradoxically, what would be really worth evaluating - the growth of understanding original creative thinking - substantially escapes the present evaluating procedures. Any sufficiently exact evaluation is restricted to formal skills, solutions to pre-determined categories of problems, retention, etc. Consequently, too much stress on evaluation entails a shift towards less important aspects of the work."

Because education is a complex process, resulting in equally complex and often subtle outcomes, it is not unusual for writers to cite the existence and importance of these outcomes, and at the same time to suggest that the intangible outcomes of education are difficult or even impossible to measure (see R.L. Ebel, 1963). Contrary to this common opinion, our research has led us to believe that all important outcomes of education are potentially measurable. Of course, this certainly does not mean that all the important outcomes of education can be measured easily. As Ebel points out: "The practical limitations of effective educational measurement are real and many, as anyone who had labored to improve educational measurements can testify. But they are not fixed eternally by the nature of man, nor is it useless to try to overcome them. The possibility of measuring the degree of attainment of all important outcomes of education does exist."

5.3 The Structure of the three Evaluation Studies

5.3.1 The Budapest Study

This study was designed to fulfill three major objectives:

(a) To answer some questions raised by the administrative educational leaders and parents.

Under this title we had to answer three questions:

i) Are the pupils in the pilot classes at least on the same level in the traditional subject matter (which they learn during a small fraction of their time) as the pupils who learn the same material for the entire school year?

ii) Did the pupils in the pilot classes gain any knowledge of such "new" topics as sets, logic, functions, etc?

iii) What is the price of any advantage in learning, in terms of such factors as pupil fatigue?

(b) To answer some questions raised by the teachers, educators, and psychologists involved in the experiment.

Everyone involved in this experiment was delighted with the "personal aims of mathematics teaching" as stated by the project: "towards the use of mathematics learning to contribute to personal fulfill", or as Dienes expresses it in his Building Up Mathematics (1960), "building up personality". We can agree with him that "it is clearly the educator's business to ensure that children get every chance of achieving as full a degree of integration as they are capable of, just as it is his business to ensure that children reach their fullest potentialities in the intellectual sphere" (Dienes, 1960; p. 25). The question the teachers, educators, and psychologists wanted answered was whether the OPI method had any success in developing the students' personalities in both cognitive and orectic regions (Dienes, 1959); did it affect, for example, the students' learning ability, creativity, flexibility, or their attitude towards the mathematics lesson, their anxiety or their achievement motivation?

To make this distinction between the two sets of questions does not mean that the teachers, educators, and psychologists were not interested in the students' knowledge, not that the administrative educational leaders were not interested in the students' personality development. It means only that to find the answer to the first type of questions is mainly an external necessity, while to find the answer to the second type of question is mainly an internal necessity.

(c) To start building an instrument sensitive enough to measure the psychological effect of this new kind of learning situation.

We hoped that while we were trying to find the answers in (a) and (b) we would collect testing instruments which could serve as a basis for future evaluation processes. In this sense we wanted to treat our data not only as the examination of the experimental effect, but also as the examination of a new evaluation battery.*

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* For more information on the Budapest study and its results, we may refer the interested reader to Klein (1972a, 1973c, 1973d).

5.3.1.1 Organization

School

For this study we chose a school in Budapest in an environment of below average socio-economic level.

This school joined the scheme in 1965 with a class of first graders (6+ years of age). This class was in grade 5 at the time of the evaluation and four other pilot classes were in grades 4, 3, 2 and 1, respectively, each having started with the experimental method in grade 1.

Classes

We used the grade 2, 3, 4 and 5 pilot classes as experimental groups and four parallel classes from the same school as control groups. All the tests were done by every student, except the measures of fatigue, which, because of the limited time, were performed by only twelve randomly selected students from each class.

In practice, because of illness and some other reasons we lost a few students in most of the tests, which caused slight differences in the number of persons taking part in the experiment test by test. Because some of the tests were inappropriate for younger children, they were carried out only in grades 3, 4 and 5 and one (the measurement of learning ability) only in grades 4 and 5.

The teachers both in the pilot classes and in the control classes were "common soldiers" of education. They teach every subject (except in grade 5), not only mathematics. None of them is specially qualified to teach mathematics. They were rated by their superior as "average teachers". To be a "good teacher" has different meanings in the traditional and in the experimental teaching conditions: e.g., the autocratic teacher who is rated superior in traditional conditions could fail in the experimental ones, and the "democratic" teacher who is rated poor in traditional conditions could be very successful in the experimental ones.

Testing Period

The measurements of fatigue were taken at 8 a.m. (before the mathematics lesson) and at 9 a.m. (after the mathematics lesson). Every other test was given only once, between 8 a.m. and noon. The entire experiment was carried out from September to December 1969.

Matching

The students in our study were not selected in any way, so we have supposed that the experimental and control classes were equal on each of the parameters at the beginning of their studies. Because all the classes were chosen from the same school milieu, we can suppose that all differences found at the time of the examination are due to the only respect in which their school environment differed, viz., mathematics teaching.

We examined this assumption on only one parameter, by comparing the results of the experimental and control groups on a "traditional" intelligence test, the Hungarian version of the Stanford-Binet test. According to the Mann-Whitney test, none of the differences between the experimental and control classes were significant at the 5% level.

In recent years there have been some interesting experiments to explore the relationship between the learning of modern mathematics and performance on intelligence tests.* If, as some of these experiments suggest, intelligence test performance is boosted by experience of modern mathematics, then parity in performance after experimental and control treatments would imply that the intelligence level of the experimental group had originally been lower. In this case, results which did not favour the experimental group would be equivocal.

5.3.2 The Sherbrooke study

The objective of the Sherbrooke study was to build a set of instruments sensitive enough to demonstrate the various expected effects of the extensively-used Dienes method of mathematics teachings, and more generally, to build a set of instruments capable of measuring some of the human factors which are so much affected by a pedagogical system. As an immediate consequence two practical tasks had to be completed:

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* Investigations of this relationship have yielded contradictory results: Hull (1963) and Fogelman (1968) reported significant differences between the experimental and control classes using the Lorge-Thorndike test. Williams (1972) found no significant differences using the same test (except in one subtest), nor in the N.F.E.R. Picture Test (Williams and Fogelman, 1972). Fogelman (1968) and Williams (1972) did not find significant differences using the Raven Matrices (except the latter in one subtest). We hope to carry out further investigation of this topic in the near future, but, to date, results neither support nor contradict our use of the traditional intelligence tests as a matching variable.

1. The selection of standard tests, among those which are available, appropriate to the expected outcome of the Dienes method.
2. The adaptation of some standard tests, and the development of some new tests, designed to measure important outcomes in the fields where standard tests are not available.

The method chosen to carry out the two practical tasks consisted of a kind of combination of evaluation and test development effort. First, traditional standard tests were employed in the evaluation process to find differences between experimental and control classes. Second, these same tests and the differences found with them were used to develop the construct validity of the new tests. Thirdly, these new tests were incorporated into the evaluation battery. To break out of a seemingly vicious circle, one of the new tests was validated on grade 5 children at Cedar Falls, Iowa, U.S.A. as well as in the Sherbrooke schools. The design of this study is quite similar to that of the Budapest study mainly to facilitate the comparison of results. Recently, cross-national education has become a topic of general interest (see e.g., Foshay, 1962; Husén, 1967; 1969; Miel and Berman, 1970). Because the Dienes method is already used widely throughout the world, it seemed natural to further develop an instrument which could be used in several countries.

5.3.2.1 Organization

The testing was carried out between May 11 and June 21, 1972, except for the achievement tests, which were performed in April and May, 1973. It included one class each from grades 3, 4 and 5 (whose students were completing their third, fourth, and fifth years with the Dienes method, respectively).

Finding the control classes was not as easy as in the Hungarian study, since there were no classes unaffected by the experimental method in the same school. In order to choose control classes from another school, experimental and control classes had to be matched in all other aspects than the one we wanted to study, namely, the learning of mathematics. The methods for matching the control classes were the following: First, the Commission des Ecoles Catholiques de Sherbrooke was asked to name the schools which they felt used similar teaching techniques and facilities, and in which the students had similar socio-economic backgrounds to those in the experimental schools. From the list provided by the Commission, we made what we believed to be the best choice possible. Our choice was verified in only four factors:

i) Social composition: The children were divided into four groups according to the social status of their parents, and X^2-tests were computed to see whether the distribution of the students in these four categories was the same in the experimental and control classes in each grade separately (see Table 5-3).

ii) Intelligence level: We used Lorge-Thorndike Intelligence Test scores obtained when the students began grade 1 and used these as a measurement of their original intelligence status. Their t-tests were carried out to see whether the average intelligence of experimental and control children was different or not at the time when they began their schooling.

iii) Conceptual level: The students' conceptual level was supposed to be measured by a test developed by Hunt (1970) and adapted to the Quebec population by us. This measure was planned not only to match experimental and control classes in this factor, but also to provide a "stratifying variable" (see 5.4), that is to arrange the students into low, average, and high conceptual level groups in order to examine the effect of the Dienes method in the three groups.

Because of technical difficulties the students' answers on this test were not scored.

iv) Teachers' attitude: An attempt was made at matching teachers' attitudes, using the Minnesota Teacher Attitude Inventory (Inventaire Minnesota des Opinions de l'Enseignant; Cook et al., English 1951; French 1972). Matching in this factor was not entirely successful.

5.3.3 The New York Study

The New York study consisted of a project with the objective of making an evaluation between two schools using the Dienes method of teaching modern mathematics at the primary level, but in two different cultural and language backgrounds as provided by the Ecole Eymard in Sherbrooke and the Fleming School in New York.

The original plan had been to evaluate the various expected psychological "side effects" of the Dienes method of teaching modern mathematics according to the pattern followed in the Sherbrooke and the Budapest Studies. Unfortunately the permission to examine children from the control school was withdrawn at the last minute. Because of this, the evaluation changed its direction: Instead of looking for differences caused by the different educational environment, we were now looking for differences caused mainly by the different socio-economical, cultural and language backgrounds. With both schools using the Dienes method, we also intended to compare the success of this method in the two schools.

5.3.3.1 Organization

Only aspects concerning the New York School are dealt with here because comparison with the Sherbrooke School is based on data collected within the Sherbrooke Study.

Testing Period

The testing was carried out from April 10th to April 20th, 1972. Those tests which needed a higher concentration (knowledge in modern and traditional mathematics) were administered in the earlier periods of the morning or afternoon sessions.

Classes

The children examined belonged to two classes at the grade 4 level and two classes at the grade 5 level which had been affected by the Dienes method for two and three years respectively. Those children who entered the school after 1970 were left out.

5.4 Summary of Results*

5.4.1 Budapest Study

Our results showed that the experimental children were superior to the control children in all the hypotheses tested except one, the greater concentration of attention. Here there was no significant difference.

Looking at the other hypotheses, the students in the experimental classes showed at least equal knowledge of "traditional" mathematics to that of those in the control classes, while the experimentals had in addition a fairly considerable knowledge of "modern" maths. This difference in the amount of information the students absorb in one mathematics lesson does not cause greater fatigue. In fact there was some evidence that the experimental method is less tiring. (This statement was supported not only by the objective measurement of fatigue, but also by the semantic differential method in which the mathematics lesson was rated as less tiring - or more refreshing - by the students in the experimental classes.)

We found significantly better results (at least in grade 5) in the experimental class on the learning ability test as well as a higher level of creativity. The gain in creativity was accompanied by a loss of rigidity in all the grades examined. A greater agreement was found in the students' judgement, and a more favourable attitude especially towards the mathematics lesson was measured in the experimental classes. The examination of the students' anxiety showed that (at least in grade 4) the students in the experimental class were less anxious, and that they had a greater need to achieve.

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* A summary of tests administered is given in Tables 5-1, 5-2, 5-3 and 5-4, Appendix 3.

In the whole process of evaluation some serious mistakes were made: Technical errors (as in the measurement of finger tremor), organizational problems (as in the measurement of modern mathematics knowledge) and even methodological mistakes (as in the measurement of the concentration of attention mentioned above).

5.4.2. Sherbrooke Study

In spite of the greater effort expended on this study the differences that emerged were less clear-cut than in Budapest. Two possible reasons might account for this discrepancy:

i) Choosing the experimental and control groups from different schools may have increased the "statistical noise," i.e. the disturbing factors.

ii) Whereas in the Budapest Study the control classes were genuinely "virgin" to the Dienes-type methods of learning these methods had penetrated into almost all the Sherbrooke schools by the time the evaluation was carried out, with the result that our control classes were probably affected to some degree.

Looking at details it appears that the two tests which failed to produce any significant differences were Demangeon's Anxiety Test and Zazzo's Perseveration Test. The Learning Ability Test (Secret Writing) also failed to reveal a significant difference between the experimental and control classes, though in this case grade 5 children showed a significant gain against the children of grade 4 in the experimental but not in the control classes. Again, Osgood's Semantic Differential Method for measuring attitude towards the mathematics lesson and Cattell's Children's Personality Questionnaire gave mostly controversial results.

The children in the experimental classes proved to be less competent at traditional mathematics (Iowa Tests of Basic Skills), especially in the part of the Test which stresses computational aspects.

On the other hand the children in the experimental classes gained a great deal of knowledge in modern mathematics; their creativity became distinctly higher than that of the control groups (Torrance Test of Creative Thinking), and they developed a positive attitude towards facing new cognitive problems rather than avoiding them (Sherbrooke Problem Facing Questionnaire).

These gains are regarded by us as highly important, and it is our intention to look for similar gains in other directions in the future.

5.4.3 New York Study

The New York Study was originally planned with the objective of making an evaluation of the various expected psychological "side-effects" of the Dienes method of teaching modern mathematics. The intention was to test and compare children from an experimental school using the Dienes approach with children from a control school which resembled the experimental school as closely as possible in all respects except the approach to mathematics, which was to be traditional.

Unfortunately the permission to examine children from the control school was withdrawn at the last minute. As a result, the evaluation changed its direction. Instead of looking for differences caused by the educational environment (i.e., the approach to mathematics) we looked for differences caused mainly by the different socio-economic, cultural and language backgrounds. Our (unchanged) experimental school was a private school of socially privileged children in New York, the Fleming School, and for our new control school we took a public school in un underprivileged area of Sherbrooke, the Ecole Eymard.

The study was undertaken at the request of the principal of the Fleming School. Through her generous co-operation the Centre de Recherches was in turn able to obtain much information of value.

Five questions in particular were of interest:

I. What is the level of knowledge in "traditional" mathematics" of the Fleming School children in comparison with those of the Ecole Eymard ?

II. Is there any difference in the achievement in the field of "modern" mathematics between the children of the two schools ?

III. What is the attitude of Fleming School students towards their mathematics lessons and their teachers ?

IV. What is the position of Fleming School children in their attitude towards problems in comparison with students in Sherbrooke and in Iowa ?

V. How well are the children of the Fleming School doing on a test which is designed to measure learning ability in comparison to those of the Ecole Eymard ?

These constituted an expansion of two questions originally posed by the Director of the School:

1. What could be said about the children of the Fleming School, on the strength of objective measurement, concerning aspects other than their intelligence and achievement ?

2. How did the standard of mathematics compare with that of the Ecole Eymard in Sherbrooke, which maintained such close relations with the originator of both programmes ?

5.4.3.1 Organization

The testing was carried out between April 10th and April 20th, 1973. The children examined belonged to two streamed classes at the grade 4 level and two streamed classes at the grade 5 level. These had been exposed to the Dienes approach for 2 and 3 years respectively. The sample tested consisted of 33 boys and 45 girls, after children who only entered the school after 1970 had been excluded.

5.4.3.2 Tests administered

Owing to the short duration of the testing period and the unexpected withdrawal of the control school in New York we were only able to administer 5 tests, as against 11 tests in the Budapest Study and 9 tests in the Sherbrooke Study.

I. Two Iowa Tests of Basic Skills (Mathematics Concepts and Mathematics Problem Solving).

II. An Achievement Test in Modern Mathematics based on Items from the Sherbrooke Item Bank.

III. A Semantic Differential and Attitude Measurement Test (after Osgood).

IV. The Sherbrooke Problem Facing Questionnaire.

V. Secret Writing: A Test of Information Acquisition and Creative Application.

To supplement our results we were also to obtain the IQ scores of the students based on the Otis-Lennon Test from the school records.

5.4.3.3. Overall results

On the basis of our observations and the 5 tests administered we can say that the children of the Fleming School seem to be happy in their work and have made good progress in both traditional and modern mathematics. Looking at the situation in greater detail it appears that:

1. In traditional mathematics the Fleming School students are without doubt superior to the students at the Ecole Eymard.

2. In modern mathematics they are marginally better.

3. The attitude towards the teachers and lessons is positive at the Fleming School.

4. The attitude of Fleming School children towards problems, in terms of their class means, is rather unexpected, one for which the evaluators have found no plausible explanation to date.

5. The standard of learning ability at the Fleming School is high compared with that at Sherbrooke.

To interpret the differences between the two schools as being due to differences in socio-economic level, intelligence level and cultural-linguistic background seems a reasonable assumption, but one which it would desirable to cross-examine by a series of further tests at some future stage.

5.5 Conclusion

In the course of more than four years and at the cost of innumerable working hours, three evaluation studies were carried out in two different continents in three different countries. A considerable amount of manpower was used for planning, developing and adapting tests, also for testing the children, scoring answer sheets and preparing statistical analyses. To this must be added the large amount of money required for the purchase of materials and the operation of machinery. Last but not least, running the tests disturbed the teachers and children in their work on many occasions.

This makes it imperative that we ask ourselves in all sincerity whether the effort is justified by its results. Unfortunately no ready yes-or-no answer is forthcoming. Our three studies had a particular as well as a more general objective. On the particular side we wanted to produce answers to the many questions which the children, parents, teachers, educators, psychologists and administrative leaders involved have posed concerning the very interesting and important pedagogical experiment called the Dienes method. On the general side we sought to develop a set of instruments sensitive enough to measure the psychological effects of any new kind of learning situation.

As far as our particular objective is concerned we are able to provide some answers: The children in the Hungarian experimental classes were not more tired than those of the control classes; the children in the New York experimental school, while still possessing good knowledge of the traditional topics, gained much knowledge in the ever-growing and important field of modern mathematics. In other directions such as the children's powers of generalization or their creativity, our conclusions are far less definite. The usefulness of these answers must be left to be judged by those who asked the questions. So far the most favourable verdict has been given in Budapest, where our investigations have helped teachers in their everyday work and the Ministry of Education has given the OPI Mathematics Project its full support, to develop into a nationwide scheme.

Our general objective has not been attained and may, in the opinion of some co-operating psychologists, be unattainable because of the magnitude of the tasks involved. We were unable to construct the instruments as projected, but if we can convince the reader that the task of searching for their construction remains an urgent one then our effort will have been worth while.

It is our intention to continue with our work in educational evaluation, and we believe that the construction of new tests especially designed for the evaluation of a particular project is a useful tool in this work. We are equally aware that the task we have set ourselves is challenging as it is ardous. If the reader shares our awareness and feels that he can offer suggestions for help let him rest assured that these will always be gratefully received.

Chapter 6

What of the Future ?

The account of the previous chapters quite naturally leads us to the following question: Granting the success of the new approaches and the validity of their underlying principles, what are the main obstacles that lie in the way of their adoption on a large scale ?

6.1 Initial difficulties

Teachers, principals and school administrators always have to face the problems arising from the implementation of new ideas. In the first instance they must be eager to adopt new procedures as individuals. But this is hardly sufficient. Innovation involves the co-ordination of countless operations ranging from teacher training to successful petitioning of treasurers for funds. If a single link is missed in the chain wastage and frustration result, and the opposition has a chance to grow stronger.

The experiences gained from the three projects leads one to suspect that from the side of the children's personality and social background there is little resistance to be feared. Of course the children in these projects were drawn from the first six years of the school system and in many cases had the opportunity to grow up with the Dienes type mathematics from their first day in school. Older children will have more firmly established attitudes - positive or negative - towards traditional approaches in mathematics teaching and might not always prove equally co-operative. But then it is unlikely that in any systematic development of the Dienes programme one would start with children who have already left the elementary school. The policy would rather be to make it possible for children already familiar with the new programme to continue with it when they change their school in adolescence. At this level the programme is not as yet sufficiently developed to permit anything definite to be said about its content or impact upon the learner. What does seem evident is that under present-day conditions of mathematics teaching a substantial body of adolescents give up mathematics altogether during their middle school years in frustration and ignorance. No project that seriously aimed to combat these undesirable outcomes could possibly do worse.

6.2 Teacher background

A far more serious obstacle is caused by the predilections of the teachers. The efficient running of a Dienes type classroom demands, among other things, a high level of mathematical competence from the teacher. But the training of the elementary school teacher - the very teacher who is to give young children their first taste of mathematics - is so diffuse in scope as to make the attainment of high mathematical competence extremely difficult. On the time table of most teacher training institutions there simply is not room to allow the student to gain a thorough grasp of all the elementary mathematical structures, let alone to struggle with some higher mathematics for its own sake. As an immediate and natural consequence many teachers take the well-defined subject of mathematics with some degree of uncertainty into their classroom, and in their professional environment they rarely find the opportunity to advance to greater certainty through the casual contact with books or colleagues. Their limited background makes them shy of trying new approaches even when a demonstration at a conference may have convinced them beyond all doubt that thought-demanding investigations with structural materials are the right procedures for getting children involved and making them eager to persevere. The only immediate solution offering itself is the provision of systematic inservice instruction to the required amount. Even for this to succeed the teacher must agree to let himself be improved, possibly at a considerable cost in time, money and effort to himself. If he accepts this he may still find that his zeal is not shared by his colleagues and his class will be the only one marching in step. The adoption of the Dienes approach by only one class would be of a very limited benefit to the children in that class or to the school as a whole.

6.3 An example

Just over a year ago the decision point for innovation was reached in an elementary school in the suburbs of Montreal, some of whose teachers had visited the Research Centre at Sherbrooke during the previous spring. Dr. Dienes and his assistants were invited to visit the school one day each week and to introduce all the 6 age levels to his approaches. During the morning they made their rounds of the classes and started the children off on games and investigations of the kind that has already been described. The children responded enthusiastically and the teachers were caught in their excitement. So much for half the day. During the afternoon the children were taken over by a group of relief teachers. This allowed the regular teachers to meet with Dr. Dienes' group in the school library and re-examine the demonstrations of the morning in an adult setting. With no classes to worry about they would hear a certain amount of exposition and then work with the structural materials themselves. Sometimes they were anxious to re-assess the children's response to a new topic. A small group of children would then be released from their classroom and came to perform under observation in the library.

The growth rate of the project was impressive. Within as little as ten weeks one noticed the increased confidence and understanding in the adults. Many materials which could not immediately be purchased were made by the children, the teachers and even the parents. Games, charts and other interesting objects of the teachers own devising appeared in increasing quantity. Clearly the fear of mathematics as something unspeakably difficult was gone. As teachers and their children rushed to discover and master new ideas there was no time to think of what they had failed to do with their mathematics in the past.

Luck, it must be admitted, had something to do with bringing a mathematics project into this school. During the previous spring a group of teachers had decided to visit Sherbrooke, and one of the advisors from the school board also went there to look around a little later himself. The favourable impressions of the visitors must later have combined with the sympathetic attitude of the school board's director to make mathematical adventure a practical and everyday reality.

6.4 Ways of expansion

What could or rather should happen next ? The network of schools administered by this particular board is extensive. Some of them must sooner or later hear about the new work in one of their members. If, on their own initiative or on suggestions from the school board, they then wished to become adventurous one or both of two procedures could be adopted. Either Dr. Dienes would have to be assisted to extend the range and time of his visits, or some of the teachers from the first school would have to be exchanged for a considerable time with teachers from schools who were seeking help with innovation It seems that in any large scale development the second procedure would become a necessity, and that progress would be still further promoted by the range of visits which Dr. Dienes or his assistants were empowered to make. If after a period of, say, one year each participating school were prepared to exchange half its teachers for the teachers of a school of similar size then this second school would enjoy a considerable advantage when it came to starting on a Dienes project. With half the teachers possessing some knowledge the work of introducing the new methods would be considerably lightened for whatever members of Dr. Dienes' team came to assist, and in the meantime the original school might be able to reduce its demand for visits by this team.

At the end of two years there would then be two schools fully capable of working with the Dienes approach and ready to exchange some of its teachers as "plant-outs". If each school exchanged half its teachers once again it would be possible to start a project in a third school with little or no regular assistance from Dr. Dienes' team.

This simple but idealised model is intended to show the central role which the teacher must play if an innovation of this kind is to be propagated. With the proper incentives and rewards he must absorb and pass on the knowledge and skills which research has developed. Not all teachers would be willing to change their place of employment, perhaps not even for the short period of one year. This would not matter as long as there remained a sufficient number prepared to make the sacrifice. And with more schools prepared to operate the plan the chances of failing to find a sufficient number of fertilizing teachers would be reduced.

6.5 Costs

At this stage it might seem proper to pause and look at the costs involved in the start of a Dienes project in just a single school. These costs would consist of payments made to the initiating team for their weekly visits, payments made to the relief teachers who release the regular teachers for further study of the new mathematics during the afternoon, and finally of the outlay on structural materials and special publications. The weight of these costs can only be judged against the total expenditure of this or that school board and against the total number of children who could be expected to benefit from the investment. If two members of the Dienes team were each paid a mature teacher's salary for visiting five schools each week and if there were 600 children in each school the cost would amount to 1/1500 of this salary per child for the first year. In the second year and later this cost would be progressively reduced as the teachers of these children gained in confidence and self-reliance and required fewer and fewer visits from outside.

As regards the structural materials and special publications an initial outlay of 200 Canadian dollars on behalf of 600 children would form a very good start. One has to remember that many of the items purchased will be of the non-consumable kind and that the 'pro capite' cost will therefore reduce much further during each year for which these items remain intact. Unfortunately the treasurers of school boards, like their cousins in other areas of local or federal government, are inclined to take a short-term view of such figures, and worry if they look too large in any one budget. This of course amounts to applying a wrong perspective to the whole process. The education of each young person is a continuous evolution which the public purse of civilized nations is called upon to promote from the age of six to somewhere between the ages of 16 and 18. Much of the educational investment that seems worth while from the pedagogical standpoint can not yield immediate and perhaps never easily measurable returns. Add to this the fact that if a substantial number of youngsters leave school without being able to handle percentages or the decimal point their deficiency does not show up as a loss in any ledger and all possible excuses for calling innovation in mathematics education expensive are ready to hand.

6.6 Extension of the Dienes approach to the Middle School

Let us assume that a Dienes project has taken firm roots in several elementary schools in a particular area. What can we hope for its long-term effects on the children? Once they outgrow the elementary school they will, under present-day conditions, as likely as not move to a classroom where approaches of a more or less traditional type are upheld. Our evaluations lead to the assumption that work in the environment of an elementary school that operates a programme of the Dienes type has no harmful effect on the children's ability to carry out calculations and other routine operations as demanded by a traditional programme in the Middle School. Clearly this is of crucial importance, since adverse feedback from the Middle School would certainly damp down the enthusiasm of administrators to give the Dienes approach its support at the elementary levels; and it would a fortiori make them determined never to allow it to shape instruction beyond these levels. A rational explanation for the children's versatility is not hard to produce. The traditional curriculum with its emphasis on repetition and manipulation only introduces the children to a very narrow range of mathematical structures, and even this range is not as clearly exposed to them in symbols as it might be with structural apparatus. At the same time this range is also an integral part of every Dienes programme, with marked differences in the manner and sequence of presentation. If these differences are the product of psychological theory and experiment then one would expect to find that their presence results in gains instead of losses for the learner.

In saying that children who have worked for six years in a classroom of the Dienes type can do all that children from a traditional classroom can do one can easily fall into the trap of suppressing half the truth. Honesty demands an answer to another question: How much more can the Dienes children do than their peers ? In this direction formal comparisons do not seem to be made, presumably because the results are regarded as a foregone conclusion. The Dienes children would seem to have a headstart and this in itself should be a strong reason for allowing them to continue with the unconventional approach.

Of course the extension of the Dienes approach to the Middle School and beyond needs much more than a simple command. There is a sort of professional discontinuity between the elementary school and whatever education lies beyond. Teachers working at the two levels differ considerably in the nature of their training. This goes towards creating a jealousy of class difference in the academic sphere. There is little rapport between the two parties, although the child's psychological development calls for a planned transition from every stage to the next. On leaving the elementary school he must expect to land in a new and possibly inferior classroom environment with a bump.

The issue also seems to revolve around numbers. As long as children from a Dienes type elementary school arrive in the Middle School as a small minority the habits of the majority will continue to dictate the mathematics programme. The majority can not easily make up for the six years of the new approach they have missed. Consequently the minority have to let much of their rich mathematical background lie fallow and can derive from it only those benefits which directly bear on the traditional work with which they are now faced.

As a first condition for tipping the balance we thus need a spread of the Dienes approach to many more schools lower down. The moment that a majority of children arrives in the Middle School with pleasant memories of what they did in the Dienes programme we can expect pressure to be exerted by both the children and their parents to develop a project for the Middle School and beyond. Then the elaborate machinery of initiation described in the discussion of the Montreal school would have to be set in motion by the responsible authorities.

This second phase in the growth of the Dienes approach would contain no fewer problems than the first. Those who teach mathematics in the Middle School and beyond usually have a broad background in the subject themselves. This makes and enables them to understand new ideas more quickly; at the same time, they tend to be more orientated towards content and less sympathetic towards the learning difficulties of students whose natural talent for the subject is less than their own. Secondly the teaching beyond the elementary level has to look towards a complex set of objectives that are imposed on the school leaver from without. There are the entrance requirements of universities and trade schools, also the demands for mathematical competence put out by the world of commerce. As a rule these objectives can not be altered or removed on pedagogical grounds. Those who set them up are governed by the demands of the sciences whom mathematics is to serve as a tool, by the need for computer programmers in industry, or by the research interests of the universities. A deadline is set for the innovator of a new teaching programme and to this his pedagogical preferences must in some way conform.

The Dienes programme seeks to maximize knowledge of structure in great variety. If this aim is to become reconciled with the demand for a high degree of competence in particular areas of mathematics careful planning and experimentation would be needed to effect the compromise. High schools frequently adopt the policy of narrowly directing their teaching at the externally imposed objectives. This leads to reinforcement by repetition and comes to be regarded as the safest way to success. A Dienes programme, on the other hand, is built up on the view that the specific tasks facing a student in his after-school employment will be performed if he has worked with sets, numbers, geometric figures, algebra, logic and come to understand how these branches are interrelated. In practical terms this means that the teacher who embraces the latter view can delay the concentration of specific practice for a considerable time and then superimpose it with greater effect on the students' enriched background. But he must also know for how long he can maintain this delay without jeopardising the results of a final examination.

6.7 The need for research

An important factor behind the success of the Dienes projects has been the close co-ordination of the work of the teacher with the development of learning materials, both written and concrete, for his classroom. Two objectives have constantly been kept in mind, the selection of a representative sample of mathematical structures and the representation of these structures in ways that will facilitate the learner's comprehension. The meeting of these objectives clearly involves a large amount of labour outside the classroom. At Sherbrooke a team of workers consisting of mathematicians and psychologists has constantly been engaged in writing, testing and re-writing. There is no finality about the process. New possibilities always suggest themselves and call for a try-out at age levels where one might have thought that the children already had plenty to do.

If successful teaching and a viable curriculum depend on activities of this kind then any school board which has taken the decision to adopt a Dienes project for a substantial number of its schools should consider giving some financial support to the maintenance of a research team at some suitable centre such as a university institute. This team could then divide their time between classroom visits and the development of learning materials and tests, much in the way that Dr. Dienes and his assistants did during the initiation of the new project in Montreal. Teaching which genuinely aims at mind formation is a responsible and arduous task. We have no more right to expect the teacher to carry this task out efficiently without support from outside than we can expect a surgeon or dentist to invent and manufacture all the equipment he uses. While the appearance of instructional materials in published form on an increasing scale together with the interchange of teachers between schools as described earlier could do much to ease the calls on such a team for assistance it could never remove them altogether. The team will wish to try out improvements and novelties, and the teachers in their turn will welcome the opportunity to bring up those residual difficulties which only personal contact with an expert can resolve.

6.8 Conclusion

One hears much of man's progress in providing fast and efficient networks of communication for himself. Pedagogy, we contend, is also a form of communication, in fact a very subtle form inasmuch as the person at the receiving end is expected to develop his capacity for reception all the while that he receives. If mathematical competence of one degree or another is becoming an essential part of modern living then it would seem that the need for making the communication of mathematics efficient is a very pressing one. To sit through some 200 lessons in the subject each year for 10 or 11 years and emerge with an awareness of gross personal incompetence is frustrating for the individual and wasteful for the society he is about to enter.

During and after the late fifties projects mushroomed forth on both sides of the Atlantic. General dissatisfaction with the success of mathematics education was coupled with an apprehension that the Western world might lose its technological lead. Public money flowed freely to sponsor experimentation for the better. What is the situation less than 20 years later ? Many projects have died a natural death or become hopelessly truncated; others like the School Mathematics Project in Great Britain have gained in strength to the point where it looks as though they will remain permanently established. But the fruits of these innovations have not been gathered, assessed or distributed to anything like the extent we are accustomed to see in the various fields of technology. Passenger ships are going out of business because research has provided us with the long distance passenger plane. Against this sort of startling progress it is not difficult to come across schools who have never experimented with new approaches in mathematics teaching or have never even heard that such approaches exist.

We have presented educational innovation with its main and side effects in what we believe to be an honest-to-fact account. The three projects here described originated at a time when interest in public interest in improving mathematics education was higher than it is today. Doubtlessly this played into our hands and won over some school authorities more easily than it might have done at other times. Now that the fervour for change has become somewhat damped we would ask the reader to free himself - momentarily at least - from the current climate of educational opinion by which he is surrounded and consider these three projects for their intrinsic merits alone. He is dealing with an elaborate theory of learning. Of this the activities of each project are an expression. The theory deals with the apprehension of the highly abstract subject of mathematics. This has made it possible for the constructors of each curriculum to generate activities for the children in great diversity, since abstract concepts can always be represented in many different ways.

The activities thus derive from the essence of the subject matter and are arranged by the underlying theory of learning so as to facilitate the grasp of this subject matter for the learner. Herein lies the essence of the Dienes approach. We believe that children take to it because it extends a helping hand to their learning needs, needs of which they are not always consciously aware. Adults have different needs and are often less able to see the merits of the method directly. They must either go and see the children or place their faith in a text. If they do either or both then we sincerely hope that they will accept our claim to have offered more people the means to learn more mathematics more quickly and with more joy.

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Appendix 1.

Some important projects teaching modern mathematics.*

AMERICA
The United States.

rogram	Year Started	Present Range	Director & Address	Support
iversity of llinois. ommittee on School thematics.	1951	Secondary Grades 7, 8	Max Beberman. UICSM Mathematics Project. 1210 West Springfield. Urbana, Illinois 61801.	University of Illinois. United States Office fo Education. National Science Foundation. Carnegie.
dison Project	1957	Secondary Elementary	Robert B. Davis. The Madison Project. 918 Irving Avenue. Syracuse, N.Y. 13210.	National Science Founda United States Office fo Education.
hool Mathematics udy Group	1958	Secondary Elementary	E.G. Begle. SMSG. Stanford University. Stanford, California 94305	National Science Founda

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Based mainly on Lockard, 1972. For some more useful information see e.g. Rosenbloom, 1964; Lockard, 1972; Sher Lamon, 1972.

Appendix 1.

mathematics.*

AMERICA
The United States.

ent Range	Director & Address	Support	Essentials
dary Grades 7, 8	Max Beberman. UICSM Mathematics Project. 1210 West Springfield. Urbana, Illinois 61801.	University of Illinois. United States Office for Education. National Science Foundation. Carnegie.	Logical structure of mathematics; study of patterns; consistency; precise terminology. Learning throu discovery. Early verbalization discouraged. Non-graded units to be used in accordance with student background and experience.
dary ntary	Robert B. Davis. The Madison Project. 918 Irving Avenue. Syracuse, N.Y. 13210.	National Science Foundation United States Office for Education.	Discovery of structure through findi one's own solutions to problems. Discussion through careful questioni Unstructured tasks; social applicati deliberately omitted. Ungraded material intended as a supplementary program. Integration of arithmetic, algebra, and geometry
dary ntary	E.G. Begle. SMSG. Stanford University. Stanford, California 94305	National Science Foundation	Structure of mathematics. Concepts mathematics as part of the whole of mathematics not just to some sub-division. Spiral approach. New content as well as conventional topi opportunity for conventional practice and review. No strong position on verbal-operational question.

e more useful information see e.g. Rosenbloom, 1964; Lockard, 1972; Sherman, 1972; Ashlock and Herman, 1970;

/

mathematics.*

AMERICA
The United States.

ent Range	Director & Address	Support	Essentials
dary Grades 7, 8	Max Beberman. UICSM Mathematics Project. 1210 West Springfield. Urbana, Illinois 61801.	University of Illinois. United States Office for Education. National Science Foundation. Carnegie.	Logical structure of mathematics; study of patterns; consistency; precise terminology. Learning throu discovery. Early verbalization discouraged. Non-graded units to be used in accordance with student background and experience.
dary ntary	Robert B. Davis. The Madison Project. 918 Irving Avenue. Syracuse, N.Y. 13210.	National Science Foundation United States Office for Education.	Discovery of structure through findi one's own solutions to problems. Discussion through careful questioni Unstructured tasks; social applicati deliberately omitted. Ungraded material intended as a supplementary program. Integration of arithmetic, algebra, and geometry.
dary ntary	E.G. Begle. SMSG. Stanford University. Stanford, California 94305	National Science Foundation	Structure of mathematics. Concepts mathematics as part of the whole of mathematics not just to some subdivision. Spiral approach. New content as well as conventional topi opportunity for conventional practice and review. No strong position on verbal-operational question.

e more useful information see e.g. Rosenbloom, 1964; Lockard, 1972; Sherman, 1972; Ashlock and Herman, 1970;

AMERICA
The United States.

ogram	Year Started	Present Range	Director & Address	Support
ithmetic Project	1958	Elementary	David A. Page. Arithmetic Project. Educational Services Incorporated. 108 Water Street. Watertown, Massachusetts, 02172.	University of Illinois. Carnegie. National Science Foundation.
iversity of ryland. thematics oject.	1959	Grades 7, 8 Elementary	Henry Walbessev. UMMaP College of Education. College Park. Maryland 20740.	Carnegie, Corporation. National Science Foundat Funds from sale of texts
andord University. Sets and mbers Project	1959	Elementary	Patrick Suppes. Experimental Teaching of Mathematics in the Elementary School. Stanford University. Stanford, California 94305	Carnegie. National Science Foundation. United States Office for Education.
eater Cleveland thematics ogram	1959	Secondary Elementary	George Cunningham. Educational Research Council of Greater Cleveland. 614 W. Superior Avenue. Cleveland Ohio 44113.	The Council

AMERICA
The United States.

nge	Director & Address	Support	Essentials
y	David A. Page. Arithmetic Project. Educational Services Incorporated. 108 Water Street. Watertown, Massachusetts, 02172.	University of Illinois. Carnegie. National Science Foundation.	Mathematical exploration. Topics with "travel" - an adventure. Intuitive thinking: guessing; inventing; trying things out. Developing a feeling for mathematical ideas. No stress on verbalization; not a complete course of study; no grade levels. Project seeks teachable alternatives for important mathematical ideas.
8 y	Henry Walbessev. UMMaP College of Education. College Park. Maryland 20740.	Carnegie, Corporation. National Science Foundation Funds from sale of texts	Mathematics as a language; precise terminology. Properties of a mathematical structure. Unifying concepts; particular emphasis on number systems. Learning through discovery. Inductive and deductive reasoning. Verbal and operational components simultaneously encouraged. Fundamental learning processes a major concern of the project.
y	Patrick Suppes. Experimental Teaching of Mathematics in the Elementary School. Stanford University. Stanford, California 94305	Carnegie. National Science Foundation. United States Office for Education.	Concept of set and operations on sets. Precise language; mathematical laws. Logic. Relationship between set theory and foundations of arithmetic. Algebraic and geometric principles.
y	George Cunningham. Educational Research Council of Greater Cleveland. 614 W. Superior Avenue. Cleveland Ohio 44113.	The Council	Logical structure of mathematics. Mathematical laws. Discovery approach. Search for patterns and relationships. Emphasis on a continuous and systemati flow of mathematics concept formation K-12.

AMERICA
The United States.

	Year Started	Present Range	Director & Address	Support	Es
a ics and Teaching	(1958) 1961	Elementary	James H. Werntz, Jr. MINNEMAST, Project Director. University of Minnesota. Minneapolis, Minnesota 55455. Paul C. Rosenbloom. MINNEMAST, Mathematics Director. Teachers College, Columbia University. New York, N.Y. 10027.	National Science Foundation	
ee on the aduate ne in tics (CUPM)	1959	All college mathematics	Alex Rosenberg. Cornell University. Ithaca, New York 14850.	National Science Foundation Mathematical Association of America	S c
ensive Mathematics (CSMP)	1965	Grades K-12	Burt Kaufman. Director CEMREL-CSMP. 103 South Washington Street. Carbondale. Illinois 62901.	Cemrel Inc. 10646 St. Charles Rock Rd. St. Ann Missouri 63071.	
eming hool Project ew York)	1969	Grades K-6	Madame Douce Fleming Corréa The Fleming School 35 East 62nd Street New York New York 10021	None	Se

AMERICA

The United States.

nge	Director & Address	Support	Essentials
	James H. Werntz, Jr. MINNEMAST, Project Director. University of Minn[illegible]. Minneapolis, Minnes[illegible]455. Paul C. Rosenbloom. MINNEMAST, Mathematics Director. Teachers College, Columbia University. New York, N.Y. 10027.	National Science Foundation	Three specific mathematics structures: the real number system, Euclidean space, a space with measure. Concepts of an algebraic structure and a deductive science. Connections between mathematics and science; cultural and historical aspects of mathematics. Working toward a coordinated mathematics-science curriculum K-9 and undergraduate courses for pre-service education of teachers. Ungraded units. Grew out of the MINNEMATH program which started in 1958. Project director 1961-1966.
ge mathematics	Alex Rosenberg. Cornell University. Ithaca, New York 14850.	National Science Foundation Mathematical Association of America	Study & improvement of college curricula.
-12	Burt Kaufman. Director CEMREL-CSMP. 103 South Washington Street. Carbondale. Illinois 62901.	Cemrel Inc. 1061[illegible] St. Charles Rock Rd. St. Ann Missouri 63071.	Games, tapes, films and other media. Vertical acceleration through independent study. Stochastic determination of goals.
des K-6	Madame Douce Fleming Corréa The Fleming School 35 East 62nd Street New York New York 10021	None	See chapter 10

CANADA

m	Year Started	Present Range	Director & Address	Support	Esse
de ches cho- atique	1966	6-14 years	Dr. Zoltan P. Dienes. Centre de Recherches en Psycho-Mathématique Sherbrooke University Sherbrooke Quebec Canada	Canada Council Conseil National de Recherches Québec Ministry of Education	See cha
			CANARY ISLANDS		
Islands atics	1965	5-16 years	Prof. Julian Caparros Morata Diplomado en Ciencias Psicopedagogicas Las Palmas Canary Islands	Private Sources	All pos Experin superse
			AUSTRALIA		
inea Project s)	1963	5-11 years	Dr. McKinnin, Dir. of Ed. Konedobu, Port Moresby, Papua New Guinea.	Territory Administration	Adapta progra condit
			EUROPE **Belgium.**		
Belge gogie tique	about 1960	Elementary Secondary University	Professor George Papy Centre Belge de la Mathématique Avenue Albert 224 1180 Bruxelles Belgium	Ministry of Education	Develop for all

CANADA

ange	Director & Address	Support	Essentials
years	Dr. Zoltan P. Dienes. Centre de Recherches en Psycho-Mathématique Sherbrooke University Sherbrooke Quebec Canada	Canada Council Conseil National de Recherches Québec Ministry of Education	See chapters 8 and 9
	CANARY ISLANDS		
	Prof. Julian Caparros Morata Diplomado en Ciencias Psicopedagogicas Las Palmas Canary Islands	Private Sources	All possible structured materials in use Experimentation, logic & creativity superseding drill
	AUSTRALIA		
	Dr. McKinnin, Dir. of Ed. Konedobu, Port Moresby, Papua New Guinea.	Territory Administration	Adaptation of Sherbrooke type program to local New Guinea conditions.
	EUROPE **Belgium.**		
	Professor George Papy Centre Belge de la Mathématique Avenue Albert 224 1180 Bruxelles Belgium	Ministry of Education	Development of mathematics materials for all levels

Table [illegible] cont.

Program	Year Started	Present Range	Director & Address	Support	
			HUNGARY		
OPI Mathematics Project (Hungary)	1963	Grades K-9	Tamás Varga Országos Pedagógiai Intézet Budapest Hungary	Hungarian Government	Se
			ROMANIA		
The Systematic Teaching of Probability	1968	12-15 years	Dr. Efraim Fischbein Institute of Psychology Str. Frumosă 26 Bucarest Romania	Academia de Stiinte Sociale și Politićs, Bucarest	To cur psy
			THE UNITED KINGDOM		
Midlands Mathematical Experiment (MME)	1960	----------------	Mr. C. Hope MME Project Harold Malley School Blossomfield Road Solihull Warwickshire England	----------------------	Ins Pre mat
School Mathematics Project (SMP)	1961	11-18 years	Dr. Bryan Thwaites Westfield College Kidderpore Avenue London N.W.3 England	Several British foundations and industrial companies	

HUNGARY

t Range	Director & Address	Support	Essentials
K-9	Tamás Varga Országos Pedagógiai Intézet Budapest Hungary	Hungarian Government	See chapter 7
	ROMANIA		
ears	Dr. Efraim Fischbein Institute of Psychology Str. Frumosă 26 Bucarest Romania	Academia de Stiinte Sociale și Politićs, Bucarest	To add the topic of probability to the curriculum and teach it from a psychological point of view
	THE UNITED KINGDOM		
-----------	Mr. C. Hope MME Project Harold Malley School Blossomfield Road Solihull Warwickshire England	-----------------------	Insight into structure Preparation of students for uses of mathematics in industry and science
ears	Dr. Bryan Thwaites Westfield College Kidderpore Avenue London N.W.3 England	Several British foundations and industrial companies	To update the secondary curriculum and maintain liaison with practising mathematicians

Table 3. cont.

THE UNITED KINGDOM

Program	Year Started	Present Range	Director & Address	Support
Nuffield Mathematics Teaching Project	1964	5-13 years	Dr. Geoffrey Matthews Centre for Science Education Bridges Place London S.W.6. England	Nuffield Foundation
Bulmershe Project	1970	5-11 years	Peter Seaborne, Berkshire College of Education, Reading, Berks, England	local government and Ministry of Education

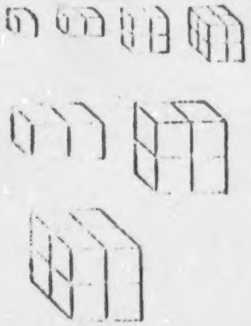
THE UNITED KINGDOM

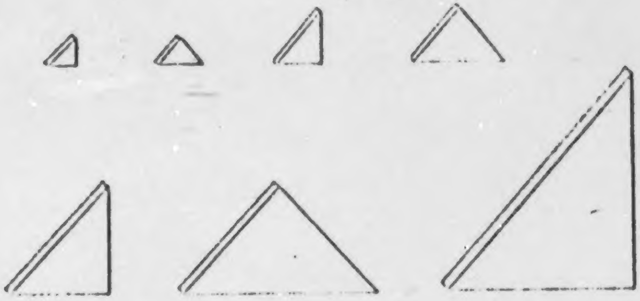
Range	Director & Address	Support	Essentials
ars	Dr. Geoffrey Matthews Centre for Science Education Bridges Place London S.W.6. England	Nuffield Foundation	A wide-ranging contemporary approach Usually materials are given to teachers who develop them for the children
ars	Peter Seaborne, Berkshire College of Education, Reading, Berks, England	local government and Ministry of Education	Adaptation of Sherbrooke type programme to English conditions.

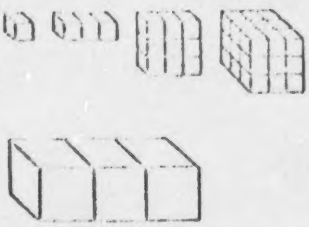
1

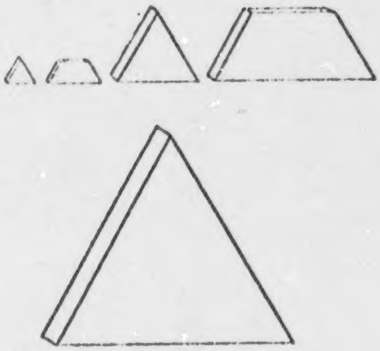
Some of the basic equipment used in the Project Schools.

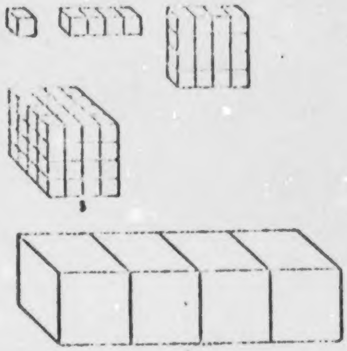
1. Multibase Blocks

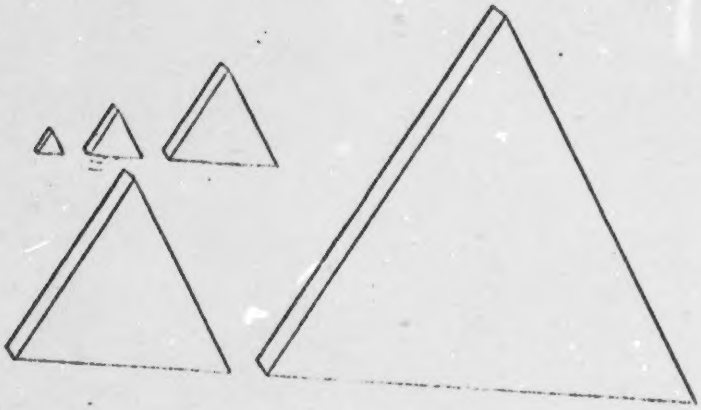


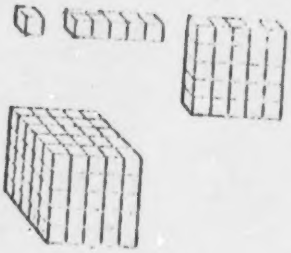


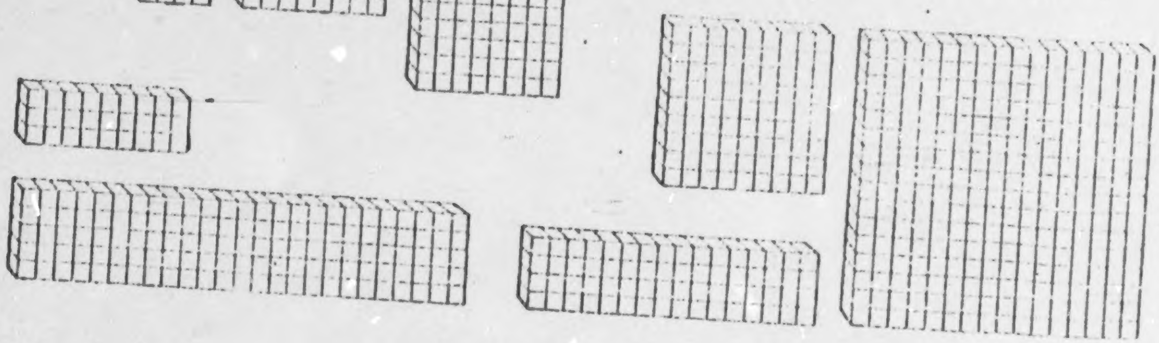












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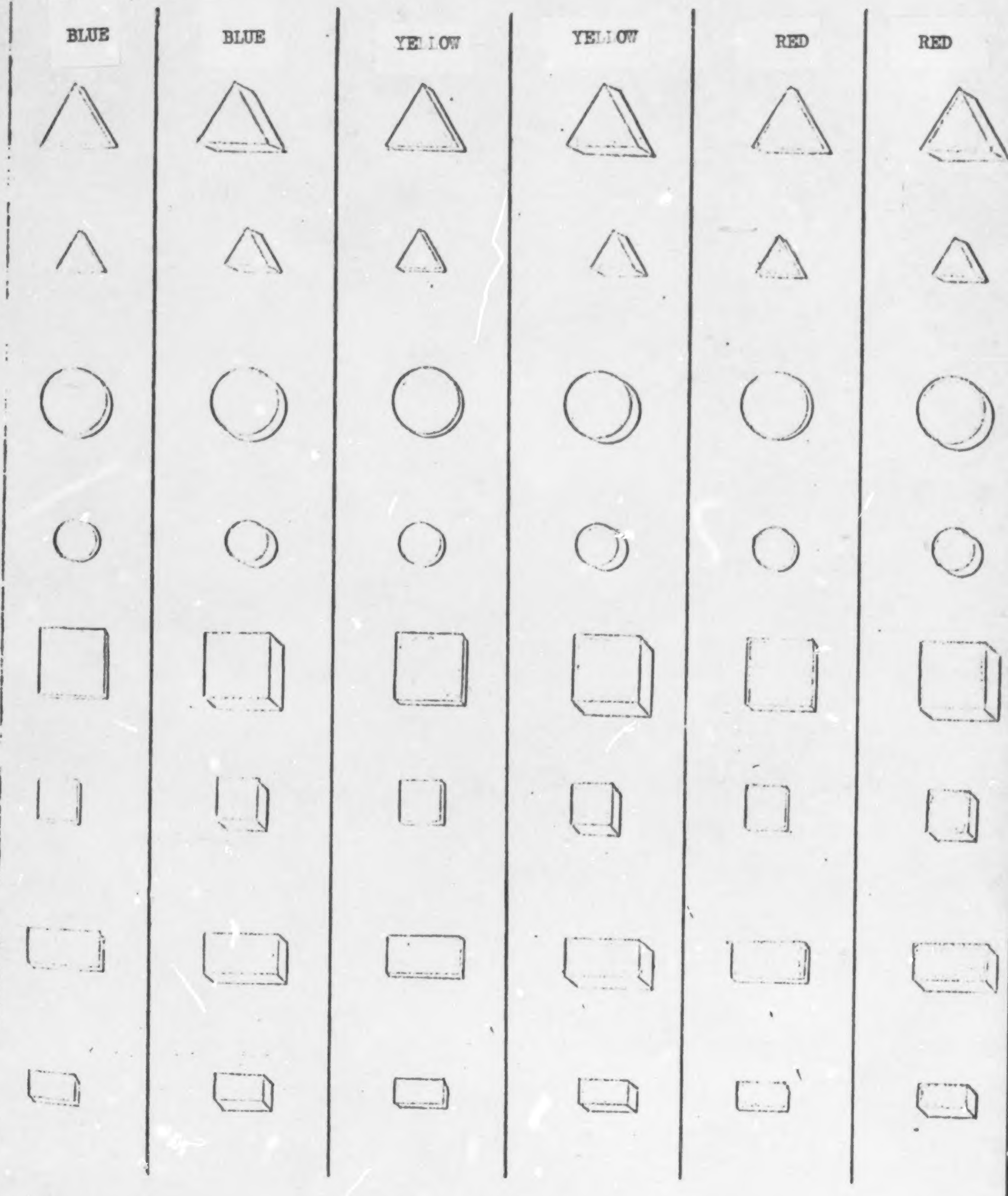
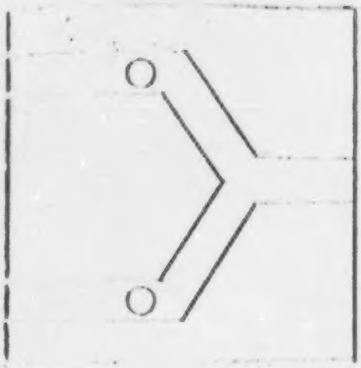


3. Logic Tracks



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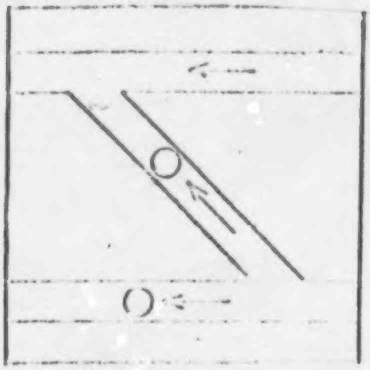
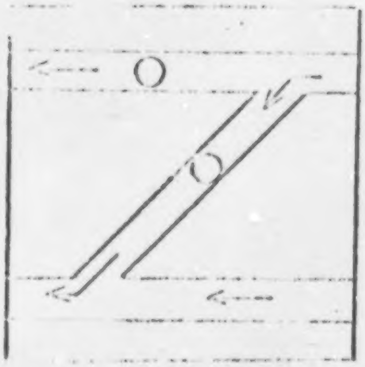


either...or



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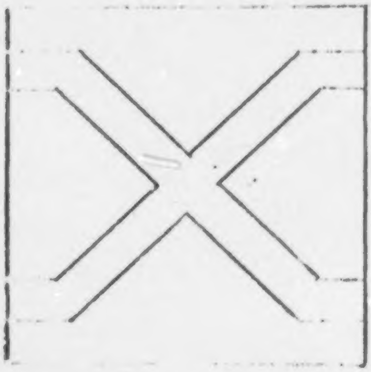

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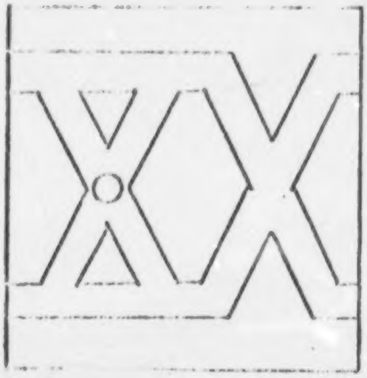

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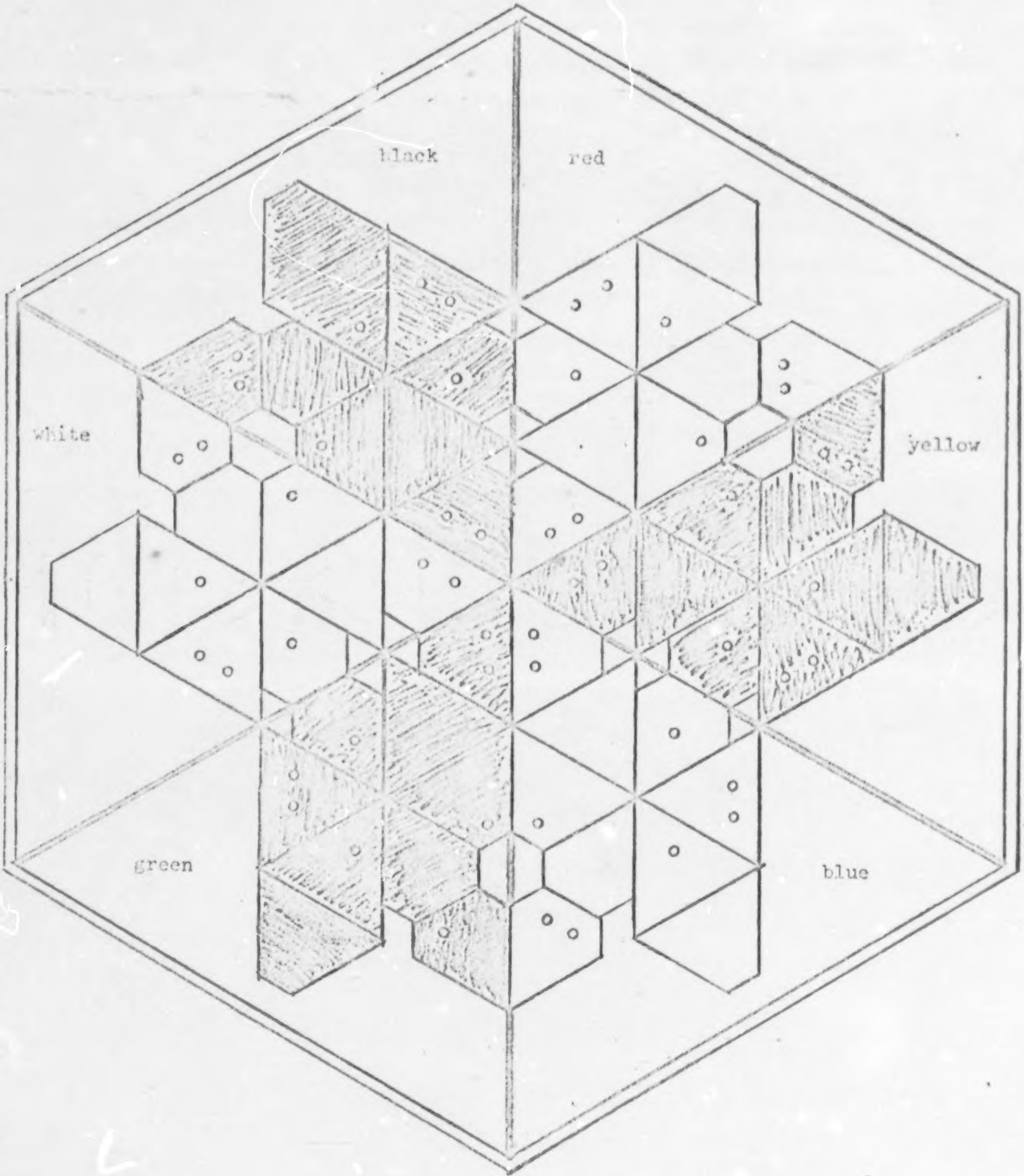
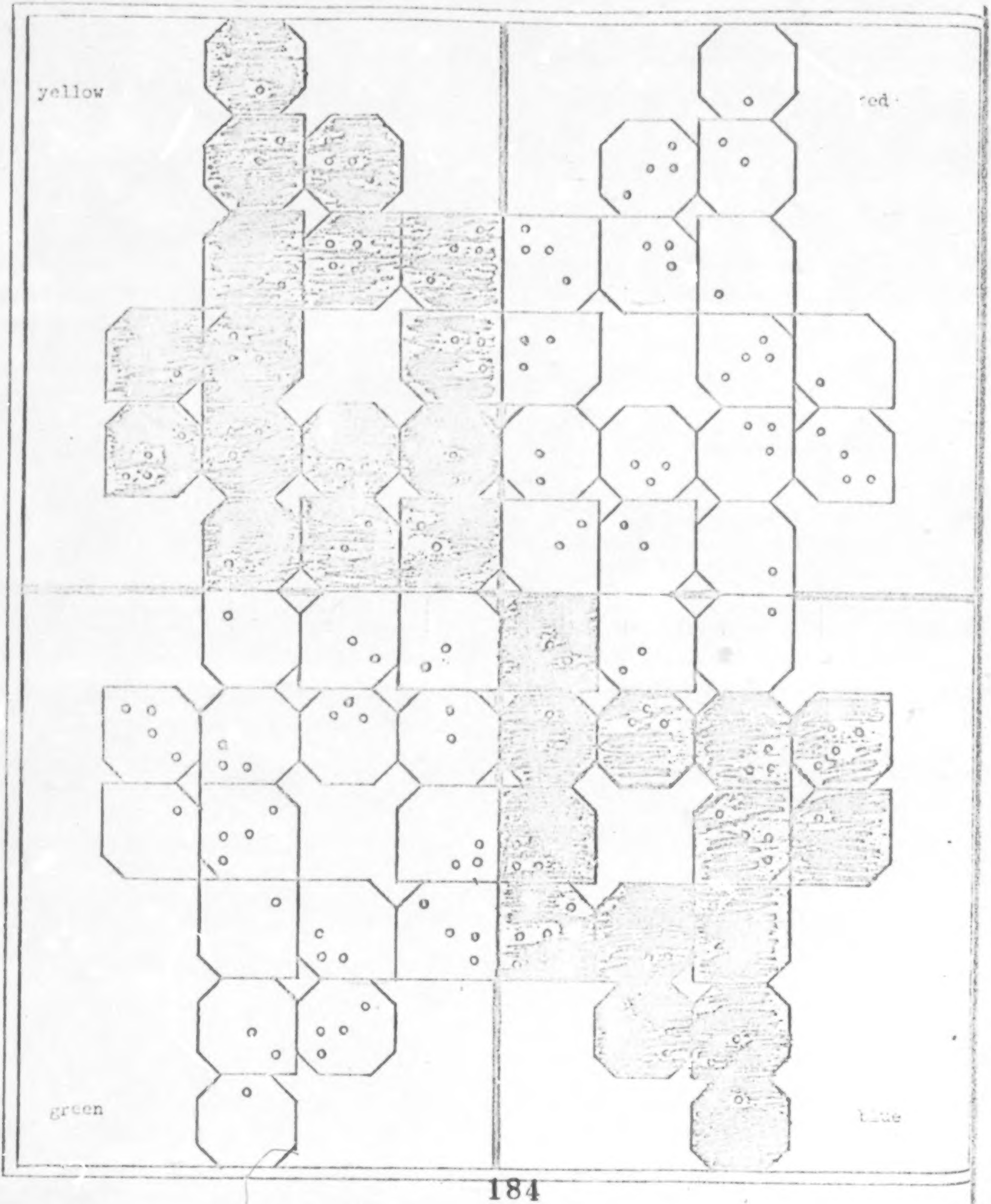
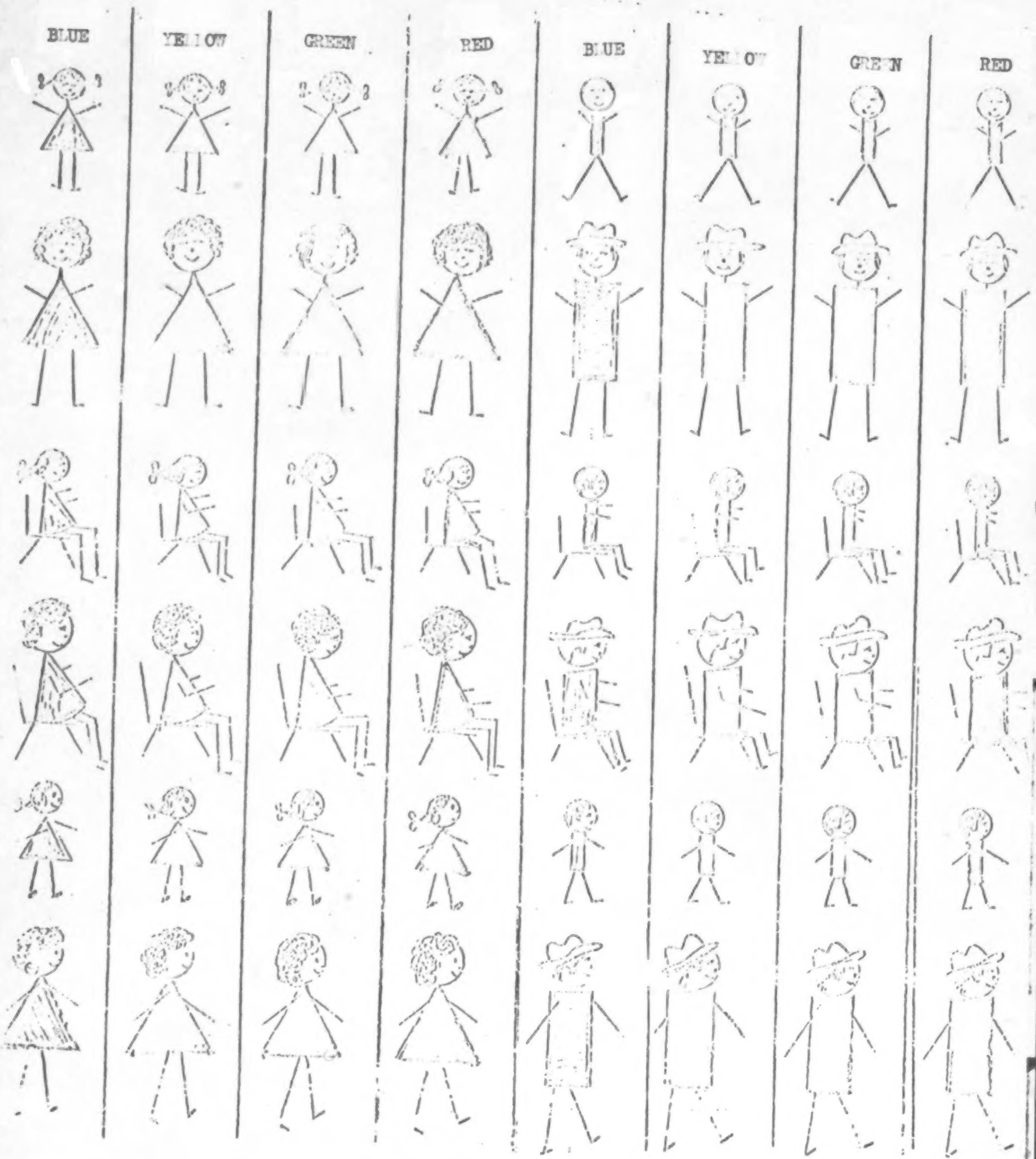
4. TRIMATHS



5. QUADRIMATHS



6. People Set



7. A List of Suppliers for the Equipment.

H.M.H., [illegible]0 ouest, rue Craig, Montréal 126, Québec, CANADA.

O.C.D.L., 65 rue Claude-Bernard, Paris 5^{e}, FRANCE.

EDUCATIONAL SUPPLY ASSOCIATION LTD., Pinnacles, Harlow, Essex, ENGLAND.

UNIVERSITY OF LONDON PRESS, Saint Paul's House 8/12, Warwick Lane, London, ENGLAND.

HERDER VERLAG Abt. Pädagogik, D 78 Freiburg, Federal Republic of GERMANY.

ORGANIZZAZIONI-SPECIALI. Via R. Franchi N. 5, Firenze, ITALY.

EDITORIAL TEIDE, Bori y Fontesta 18, Barcelona 6, SPAIN.

LONGMAN GROUP LTD., 74 Grosvenor Street, London, W.1., ENGLAND.

FRANZ SCHUBIGER, Postkonto W 111b 286, Winterthur, SWITZERLAND.

EDITORA HERDER, Sao Paulo, Brazil, SOUTH AMERICA.

MALMBERG/N.V., Sertogenbosch, Leeghwaterlaan, 16 NETHERLANDS.

FELTRINELLI EDITORE, Milano, ITALY.

LIVROS HORIZONTE, Rua de Madalena, 213, Lisboa, PORTUGAL.

le 5 - 1

Appendix 3.

An Overview of the three Evaluations.

Objective	Environment	Personality factor	Cr	
			Budapest Study	Sher
low children with a nable mastery of im- nt techniques and know- of Mathematical facts.	A by-product of activities involving different kinds of problems.	Knowledge level in "traditional" maths.	An achievement test based on the "traditional" maths curriculum Grade 2,3,4,5	Iowa Test Math Hier
eate a good understand- f basic mathematical pts (e.g. variable, ion, isomorphism, lin- , etc.) and of their -relations.	Problems of set, logic, topology, combinations, probability, statistics, groups, etc. Using the "multiple embodiment" technique, etc.	Mathematical ability.	An achievement test based on the experimental maths curriculum	An a from (Con
vent fatigue in the en as a result of the lesson.	Less external pressure to achieve; fewer restrictions. Avoidance of monotony by different kinds of tasks, materials, activities and interesting problems.	Fatigability.	Reaction to visual and acoustic stimuli; heart rate; visual field; finger tremor. Grade	
ture the child's atten-	Variety of materials and activities. Problems interesting to children.	Attention level.	Toulcuse-Pieron test. (Pieron - 1929) Grade	
elop mathematical ng ability.	"Learning centred" rather than "teacher centred" arrangement (independent, individualized learning).	The dynamic of mental development (Vygotsky, 1962) Learning ability; something determining the performance and the speed of learning in a learning task, as well as the creative application of the gained information.	Secret Writing.- A test and creative application Hungarian form (Klein, 1970).	Fre (Kl

Appendix 3.

An Overview of the three Evaluations.

Environment	Personality factor	Criteria		
		Budapest Study	Sherbrooke Study	New York Study
duct of activities in- different kinds of	Knowledge level in "traditional" maths.	An achievement tést based on the "traditional" maths curriculum Grade 2,3,4,5	Iowa Tests of Basic Skills, Form 5, Test M (Mathematics Concepts and Mathematics Problem Solving; Hieronymus and Lindquist, 1971.	
of set, logic, top- ombinations, prob- statistics, groups, ing the "multiple t" technique, etc.	Mathematical ability.	An achievement test based on the experimental maths curriculum	An achievement test based on items from the Sherbrooke Item Bank (Connes, 1973.	
rnal pressure to fewer restrictions. of monotony by kinds of tasks, , activities and ng problems.	Fatigability.	Reaction to visual and acoustic stimuli; heart rate; visual field; finger tremor. Grade	✗	✗
f materials and ac- Problems ng to children.	Attention level.	Toulcuse-Pieron test. (Pieron - 1929) Grade	✗	✗
centred" rather cher centred" nt (independent, lized learning).	The dynamic of mental development (Vygotsky, 1962) Learning ability; something determining the performance and the speed of learning in a learning task, as well as the creative application of the gained information.	Secret Writing. A test of information acquisition and creative application (IA) Hungarian form (Klein, 1970).	French form (Klein, 1972).	English form (Klein, 1973)

Objective	Environment	Personality factor	Budapest Study	She
p the child explore own his original ive" ideas. To allow ild's special "diver- hinking" to blossom.	Letting the children discover regularities. Letting the children construct tasks themselves. Giving tasks with a variety of possible solutions (indefinite tasks, open questions, etc.).	Creativity: fluency (ability to give multiple answers to the same given information); flexibility (spontaneous shift of set), originality (rarity of responses in the population) (Guilford and Hoepfner, 1971).	Items from Torrance's (1962, 1966) creativity tests.	Te Cr A 19
rease rigidity of ng.	Sensorimotor experiences with a sufficient number of varied, concrete materials. A great variety of working tools and problem situations.	Perseveration; rigidity: "A tendency to persist in responses that may previously have been suitable in some situation or other but that no longer appear adequate to achieve current goals or to solve current problems" (Wesley, 1953).	Test de persévération (Zaz	
p the children ach other's ies better.	Group activities (e.g., competitions between groups, grouping the children to find the solutions of a more difficult problem). Right to "cheat" when problems appear. Giving more emphasis to intrinsic motivation (not giving reasons for envy, arrogance, contempt, etc. which could distort the children's opinions about each other.	Mutual appraisal: the capability of evaluating other members of a group.	"Who are the 3 most clever pupils in your class?" A tailor made test.	

Environment	Personality factor	Criteria		
		Budapest Study	Sherbrooke Study	New York Study
the children disco- ularities. Letting ldren construct emselves. Giving ith a variety of e solutions nite tasks, open ns, etc.).	Creativity: fluency (ability to give multiple answers to the same given information); flexibility (spontaneous shift of set), originality (rarity of responses in the population) (Guilford and Hoepfner, 1971).	Items from Torrance's (1962, 1966) creativity tests.	Tests de Pensée Créative. Forme A (Torrance, 1972)	X
notor experiences sufficient number ed, concrete s. A great of working tools blem situations.	Perseveration; rigidity: "A tendency to persist in responses that may previously have been suitable in some situation or other but that no longer appear adequate to achieve current goals or to solve current problems" (Wesley, 1953).	Test de persévération (Zazzo and Stanbak, 1964)		X
tivities (e.g., ions between grouping the to find the s of a more t problem). "cheat" blems ap- iving more to intrinsic on (not easons for envy, e, contempt, etc. uld distort dren's opinions ch other.	Mutual appraisal: the capability of evaluating other members of a group.	"Who are the 3 most clever pupils in your class?" A tailor made test.	X	X

Table 5 - 4

Objective
To develop a positive attitude toward problems.
Help to develop the whole of the students personality.

Environment	Personality factor	Criteria		
		Budapest Study	Sherbrooke Study	New York Study
Indefinite problems. New problems each time.	Attitude		Sherbrooke Problem Facing Questionnaire. French form (Klein, 1973)	English form (Klein, 1973)
Freedom in activities; abundance in the possible activities	Reserved - Warmhearted. Less Intelligent- More Intelligent. Affected by Feelings - Emotionally stable. Phlegmatic - Excitable Obedient - Assertive Sober - Happy-go-lucky Expedient - Conscientious Shy - Venturesome Though-minded - Tender-minded Vigorous - Circumspect Forthright - Shrewd Self-assured - Apprehensive Casual - Controlled Relaxed - Tense		Children's Personality Questionnaire Form A and B (Porter-Cattell 1963).	
	Anxiety Extraversion			